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THE MATHEMATICS TEACHER

Volume XXVIII Number 2

Edited by William David Reeve

The Need for a Reorientation of Secondary Mathematics from the College Viewpoint*

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In discussing the orientation of secondary mathematics, I shall not consider explicit details about the content of high school courses or about the methodology of their teachers. Moreover, although I shall be speaking from the viewpoint of a teacher of college mathematics, I shall not restrict myself to topics which are of importance only as preparation for college work. Part of my remarks will relate to the matter of teacher morale, and will suggest inferences with respect to the mathematical attitude of those responsible for the administration of the high schools. Another section of my paper will consist of general recommendations about the placement of students in courses in secondary mathematics.

As a background for suggestions which I shall present later, first permit me to rehearse certain fairly familiar facts about the position of mathematics in the college and secondary fields, and let me commence by references to the college field.

* An address read at the joint meeting of the National Council of Teachers of Mathematics and the Mathematical Association of America at Pittsburgh, Pa., Dec. 29, 1934.

It is well known that at one time many American colleges of the first rank required not only a large amount of high school mathematics for entrance but, also, a moderate amount of college mathematics for graduation, regardless of the major field of study involved. The diversification of the curricula of some colleges and the growth of the elective system created a tendency to eliminate the general requirement of college mathematics, although *very properly* this tendency has had no effect in those colleges where the mathematical nature of the curricula or a special educational viewpoint causes mathematics to assume unique importance for all students. In the extremely large number of colleges where there now exists no blanket requirement of mathematics for graduation, it is left for mathematics, as well as for most other subjects in the curriculum, to attempt to justify itself objectively in the case of each student.

In the last twenty years, it has become increasingly easy to demonstrate this justification for college training in mathematics on both cultural and practical grounds in the cases of large numbers of students. For, at present we are witnessing an increased emphasis on the need for advanced mathematics in the physical sciences, and of at least elementary college mathematics in the development of many other disciplines not commonly referred to as mathematical fields. Thus, the tremendous growth in the use of statistical methods and other mathematical procedures in the natural sciences, where we include psychology, in the social sciences, and in the theory of education has led many students from these fields to study mathematics. As a generalization, I consider it reasonable to say that today mathematics is in a stronger and healthier position in the college curriculum than ever before.

The validity of this opinion would not be questioned by anyone so far as the remarks apply to the importance of mathematics in the physical sciences and the various branches of engineering. However, we might expect a non-mathematician to be more hesitant about accepting our opinion concerning the utility of mathematics in many of the fields not originally cultivated by use of mathematical devices. Interesting evidence to present in this connection is furnished by a committee report[1] on the "Collegiate Mathematics Needed in the Social Sciences," presented

[1] The report is published in full in the *American Mathematical Monthly*, Vol. XXXIX, 1932, pp. 569–577.

to the Social Science Research Council in December, 1931. This committee report is particularly worthy of notice in view of recent tendencies to make the social sciences exceptionally prominent in all secondary and college curricula.

The committee just referred to consisted of H. R. Tolley, Director of the Giannini Foundation of Agricultural Economics, University of California; F. L. Griffin, Professor of Mathematics, Reed College; Holbrook Working, Economist of the Food Research Institute, Leland Stanford University; C. H. Titus, Professor of Political Science, University of California at Los Angeles; and Mordecai Ezekiel, Assistant Chief Economist, the Federal Farm Board. Their report cited certain topics in college mathematics which might serve as a basis for a three-hour course for two or three semesters, uniquely fitted for a student majoring in the social sciences, even though he might not intend to proceed with graduate work in his major field. I take the liberty of quoting certain passages from this report.

". . . the Committee is of the opinion . . . that all students of the social sciences would find some work in collegiate mathematics very helpful."

"Training in mathematics is useful to those who take undergraduate courses in economic theory. . . ."

"Mathematics courses covering the listed topics would be very helpful as a preliminary to a first course in statistics. . . . The statistics course could then utilize the usual amount of time to carry the student much farther in the knowledge of statistical methods, their possibilities and limitations, or the customary ground could be covered in a much shorter time."

As a basis for argument concerning the importance of secondary mathematics, I might lend an ear to the assumption, *contrary to fact*, that the committee report just mentioned is too enthusiastic about the usefulness of mathematics. Even under such an unfavorable assumption I could state that this report gives ample support from the social science field to the *conservative* claim that a strong foundation *at least in secondary mathematics* should be a prerequisite for college work in any statistically inclined field. This claim would have particular force in the fields of economics, education, and psychology, as well as in important parts of other natural sciences and social sciences.

Some persons might be inclined to argue that remarks about the

importance of mathematics in the college curriculum are refuted by existing tendencies to remove explicit mention of more than a bare minimum of mathematics from the college entrance requirements of various important universities. Such an argument would be based on a lack of appreciation of the existing college viewpoint about entrance requirements. The underlying tendency is mainly to make such requirements extremely *flexible* as an aid to administrative procedure. The removal of explicit mention of various subjects which formerly were definitely required does *not* mean that they are no longer important. This remark is illustrated by the recent action of various universities in adding strong advice about the desirability of certain high school preparation to the bare statement of the flexible entrance requirements. For example, one major university recently inserted a statement in its bulletin listing all subjects and professional curricula which were *closed*, for major work, to a student without strong preparation in secondary mathematics. This closed list includes a majority of the subjects offered by the University, the exceptions being essentially the various fields of literature, history, journalism, and some special curricula, notably in the field of art.[2]

My remarks about the position of mathematics as a part of the college curriculum might be summarized by saying that, in the college field, we are witnessing a very successful, even though informal *offensive* action on the part of mathematics, where fortunately the soldiers fighting to enhance the importance of our subject are, in large numbers, volunteers from outside our field.

On turning to a view of mathematics in the secondary field, you know that we find a decidedly unfavorable picture compared to that which I have sketched for the college field. We may agree that in a major portion of the United States we find what may be termed an *offensive* action against mathematics in the secondary schools. One somewhat theoretical basis for this anti-mathematical attitude is centered around debatable psychological questions as to the ease of transfer of training from one field to another. This method of attack against mathematics, although successful in some battles in the past, is now meeting what I consider to be a decidedly successful counter attack from the most desirable direction—the field of psychology itself. I shall leave this point of theoretical psychology for others to debate. Let me merely state my personal opinion that the high standing of many psychologists,

[2] See the forthcoming bulletin of the University of Wisconsin.

who advance views favorable to the notion that transfer of training occurs, leads me to hope that before long the field of psychology will have completely discredited the attitude with respect to transfer of training which in the past has been a basis for anti-mathematical propaganda.

As an additional important basis at present for actions against secondary mathematics, we are struck by the admitted fact that the complexion of the student body at the high school level has changed even more radically than at the college level during the last twenty-five years, and with added acceleration during the last five years. Thus, we learn that the registration in grades nine to twelve has increased from approximately 1,100,000 in 1910 to about 6,000,000 pupils today. Hence, in contrast to earlier conditions, a larger percentage of the population is receiving instruction through the twelfth grade and the resulting student body presents a much wider range than before with respect to intelligence, cultural background, and future interests. Naturally, it would be reasonable to agree that such conditions, particularly in the aggravated form they have assumed during the depression, might justify departures from the mathematical viewpoint which existed in the secondary field during the preceding generation. A major question, however, is "what changes would be justified?"

I should prefer to class recent tendencies for changes in regard to secondary mathematics under two headings. In a first category, I would list those changes which are due to the intelligent curiosity of teachers of mathematics and their desire to make adjustments in the light of past experience and anticipated future needs. For instance, in this respectable first category I would list experiments on unified treatments of various formerly separated parts of high school content; also, I would include here the efforts to adjust text books and class procedures to the individual differences of students by the use of carefully organized optional material.

In a second and undesirable category I would list those changes which are the result of prejudice, debatable psychological theories, or the desire to follow the line of least resistance. In this category I would include the existing movement in many schools to restrict the opportunity for high school students to take more than a bare minimum of mathematics, either by omitting the regular presentation of advanced mathematics or by exposing capable pupils to advice which encourages the replacement of advanced

mathematics by other less defensible material. Also, I would include here the tendencies to reduce the intellectual level of the whole ninth-grade course to the plane of the weaker students, now present in great numbers in most high schools.

I do not propose to give an exhaustive list of present good and bad tendencies. Hence, permit me to close explicit mention of them by admitting the existence of various bad tendencies, many of which are induced by effects of the recent tremendous increases in high school enrollment, or by the well meant, although sometimes unfortunate, actions of administrators of the schools in their efforts to effect economies, or to inaugurate changes to match the observed trends in our social order.

Let me summarize the background I have now sketched. In the college field and in the intellectual world in general we witness mathematics in a strong position, where there is increasing demand not only for the cultural and practical benefits of advanced mathematics but, also, an independent demand for elementary and intermediate mathematics. For special contrast with certain high school tendencies, we note the explicit usefulness of mathematics in the social sciences and other disciplines not currently called mathematical fields, which are of definite interest to the mathematical layman. In the secondary field we observe the mathematical staff endeavoring to maintain the traditional reputation of mathematics as a progressive science by intelligent evolution of the content and methodology of the mathematical courses. And, we recognize that these efforts are being carried on in the face of a tremendously increased student load, frequently under unfavorable class-room conditions and in many cases under the handicap of an unfriendly administrative attitude with respect to mathematics. In the light of this background, what can we suggest in the nature of a helpful reorientation?

My first suggestion relates to the question of teacher morale. I recall hearing a speech presented to a group of teachers of secondary mathematics where first there was given a picture of the state of affairs in regard to secondary mathematics similar to what has been sketched today. Then, again and again in the course of the speech I heard the refrain: "*Mathematics is on the defensive; you must prove that you are progressive: hence you must experiment with your content and methodology so as to remove the objections to your subject.*"

I recognize the good intent of such remarks but I should prefer to alter them decidedly. I suggest as a preferable tune: "*The teacher of secondary mathematics must assume an offensive position, and must not experiment just to show his liveliness.*" Ill conceived experiments on too wide a scale or unsupervised experiments by untrained experimenters may create grave evils. Although I distinctly favor suitable experimentation, I am opposed to the idea that educational experimentation is easy, in any field. If we should reflect unfavorably on our established methods and content by radical and far reaching experimentation which might confuse many of our colleagues and place major portions of our field in a turmoil, we might easily add fuel to flames which later might consume us.

In dealings with any interested person, I would assert that the importance of mathematics in all previous history and, in particular, the strong position of mathematics on both cultural and practical grounds today, even in relatively new fields like economics and other social sciences, justifies *increasing* rather than *decreasing* the emphasis on mathematics in the high school curriculum. Hence, regardless of the difficulties encountered by many secondary students in the study of mathematics, there is ample cause for a *reasonably intelligent student* to take a considerable amount of high school mathematics.

As a companion attitude to this viewpoint, I should then recommend that all of our experimentation should be based on the premise that, in the hands of well prepared teachers, *our basic content and methodology are good.* For instance, I should object if any experimenter should commence by damning the existing order, whose age and *marvelous* past achievements in the development of mathematics justify hearty respect. An experimenter does harm to the cause of learning if he dares to commence his presentation of facts with the bold claim that there is only *one* good way to organize content or to present it to students; human experience makes such a bold claim appear ridiculous. In approaching experimentation, I should claim that through gradual evolution our field *has* kept up with all justifiable trends in education and all changes in civilization—in fact, mathematics has built the roads to progress. I suggest sober, carefully considered experimentation, performed mainly by those few who have the specialized abilities necessary for successful efforts.

As an all important experiment for the *college* field which is associated with teacher preparation, I recommend a study of improved methods for training high school teachers of mathematics, with an increased emphasis on the *mathematical content* studied. For the present, in the secondary field, I am inclined to suggest experimentation on *methods* of teaching rather than radical alterations in the basic content of our courses for the intelligent students. Also, I favor detailed consideration of the problem presented by the large mass of students at the lower end of the intelligence scale.

Finally, not so much as an experiment, but rather as a reaffirmed principle, I believe that we should openly throw the combined weight of our influence behind all measures designed to improve the opportunities for good teaching of secondary mathematics *as it now exists*. Thus, for instance, we might be inclined to insist publicly that mathematics can be well taught to the average class only in groups of moderate size, at the secondary level, and that the importance of mathematics justifies any attendant expense resulting from the use of such moderately sized classes. And, we should forcibly announce that mathematics as a science should not be held responsible for the unhappy results which are witnessed when secondary mathematics is presented—or, rather, when *attempts* are made to present it—by football coaches, teachers of history, music instructors, or other members of high school staffs who do not even have the basis to personally appreciate the content of secondary mathematics.

I have already mentioned that there is reason to consider how the courses in secondary mathematics should be organized in view of the recent influx of large numbers of pupils at the lower end of the intelligence scale. My remaining remarks, which are not very novel, could be summarized as my personal preferences for attacking this problem. As an introduction to my suggestions, let me recall to you that I have emphasized the opinion that a strong mathematical foundation is necessary for efficient college work in vital parts of the social sciences and in any other fields which rely heavily on statistical methods, as well as in the natural sciences where the usefulness of mathematics is beyond question. This viewpoint leads me to conclude that a student should offer secondary mathematics at least through a second course in algebra for entrance to college in order to avoid restrictions and handicaps in his college career.

Another opinion which is in the background of my later suggestions is my conviction of the fundamental inaccuracy of a diagnosis, at any time before graduation from high school, that any particular student of reasonable intelligence will *not* go to college. In addition, I consider it fair to assume that we can demonstrate the desirability of well taught advanced secondary mathematics in preference to the substitute material available in the secondary curriculum, even when college contacts of secondary mathematics are disregarded. Hence, I conclude that all high school pupils of reasonable intelligence should be treated alike so far as their mathematical training is concerned. These considerations lead me to suggest the following procedure for the placement of students in secondary mathematics.

Suggestion A. Students entering the ninth grade should be divided into two categories on the basis of their mathematical ability, *regardless of their apparent prospects for college entrance.* The category of low rank, whose size I prefer not to estimate, would contain those who would reach the limit of their mathematical powers in attempting to learn the classical first year high school algebra.

Suggestion B. Students of the lower category should be segregated from those of the upper category in classes in mathematics and should study a concrete variety of mathematics in the ninth grade, and possibly also in the tenth grade, where any algebraic methods and logical procedures involved are pitched on a low plane, and where the classical courses in algebra and demonstrative geometry should not be permitted to interfere with the development of that course which is appropriate for the students involved.

Suggestion C. All students of the upper category should study mathematics through a second course in algebra. Students who then foresee possibilities for college entrance would profit by the study of additional mathematics, which should be as extensive as possible for those who plan college work in any of the intensively mathematical fields.

Suggestion D. All students of the upper category, regardless of their prospects for college entrance, would receive benefit from a semester course in applied mathematics having advanced algebra as a prerequisite, or perhaps from a wide view survey course covering parts of trigonometry, analytic geometry, and calculus.

Let me give added emphasis to only two points in the placement system I have described. I suggested the division of ninth grade students into two categories, to be provided differentiated training, in order to give appropriate instruction to the weaker students and to maintain high standards for the others. This division into categories would require a statistically determined scale of mathematical aptitude for high school freshmen and corresponding norms of mathematical achievement through the eleventh grade, where these norms should be based on *selected well taught classes*. I am well aware that some such division into categories is practiced in many high schools. Also, I realize that, in a small high school, the nearest possible approach to such a placement system might be through the teaching of *one* course to *all* students with strict attention to adjustments for individual differences.

I should also like to emphasize my suggestion about a course in applied mathematics with advanced algebra as a prerequisite. To avoid the possible misinterpretation that this suggestion places too great emphasis on the utilitarian aspects of mathematics, I wish to express the opinion that such aspects should be considered as composing only *one* side of our objectives and that the less tangible objectives relating to training in mathematical thinking and associated habits should be kept equally in mind.

With recognition of this traditional viewpoint, I return to my suggestion of the applied course as an interesting and tangible feature of the sequence of courses in secondary mathematics. This applied course could include an algebraic—not *wordy*—treatment of the theory of interest, with a simplified presentation of large parts of the current college course in the mathematics of investment; also, an elementary mathematical introduction to the theory of life insurance and pensions; also, a discussion of index numbers and other simple statistical notions with some of their applications in the natural and the social sciences. I favor such an applied course, regardless of whether or not the high school student intends to enter college, because I wish him to have as much experience as possible with real applications of secondary mathematics so that he will avoid the viewpoint that mathematics is a cloistered subject.

I have refrained from making remarks relating to the content of the standard courses in secondary mathematics mainly because

I believe that, by and large, this content is satisfactory when taught by well prepared teachers under favorable class-room conditions. However, I wish to express one opinion concerning the orientation of the content of high school mathematics. I believe that we should have prominently in mind the fact that secondary mathematics by itself is definitely useful, from both the cultural and the practical viewpoints, regardless of possible future uses of the content in college work.

Let me summarize the remarks which I have made. I have urged a militant attitude on the part of teachers of mathematics in the secondary field, with emphasis on the growing demand for training in secondary mathematics as a relatively new feature in many fields not formerly considered to have mathematical interests. I have emphasized the importance of improved content training for teachers of mathematics and improved opportunities for good teaching by the well prepared teacher. I have recommended a placement system for segregating secondary students into categories which would be given differentiated treatment, with the best category receiving at least two and one-half units of mathematics, regardless of the collegiate intentions of the students. For all of secondary mathematics, I have urged the point of view that secondary mathematics by itself is culturally and practically useful, regardless of later college contacts.

In conclusion, I wish to state that I am fully aware that much of what I have said is lacking novelty and that many of my opinionated statements are subject to strong objections, and are in no sense summaries of the reactions of college teachers of mathematics. I merely hope that my formal reiteration of some facts may have been useful.

Announcing the Tenth Yearbook

The Tenth Yearbook of the National Council of Teachers of Mathematics on "The Teaching of Arithmetic" will be ready for shipment on February 20, 1935. Price $1.75 postpaid for a bound volume. See page 1 of this issue for particulars about the other Yearbooks.

The Need for a Reorientation of Secondary School Mathematics from the High School View Point*

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THREE YEARS ago a prominent professor of education in a large eastern University published a book entitled, "Secondary Schools in 1960." The author assumes the rôle of prophet, and among his predictions for 1960 are the following: "A very few students in eleventh and twelfth grades take a course in Pre-engineering Mathematics. (But in practice most prospective engineers take *all* their vocational mathematics in professional schools—as do the followers of nearly all other vocations. . . .) In marked contrast to the earlier time, it is found that commonly not more than twenty percent of pupils take *any* mathematics in grades seven to twelve."[1]

If after twenty-five years this prophecy should prove to be accurate, those who survive to see its fulfillment may frequently have occasion for reflection upon the subject of our discussion, namely, "The Need for a Reorientation of Secondary School Mathematics." Some may think that the prophecy I have just read is an extreme viewpoint, yet in the *American Mathematical Monthly* of March 1934, we find Professor A. A. Bennett quoted as saying: "With no feeling of hostility to mathematics, the educators whom he consulted agree that the tendency is now against mathematics and that no concerted effort can save the situation."[2]

The truth is that the movement against mathematics is not a recent development. Presaged by sporadic attacks such as those of Berkeley, Schopenhauer, and Huxley, it was greatly accelerated by the decline of the doctrines of formal discipline and automatic transfer of training. Since the turn of the century, the schools of

* An address read at the joint meeting of the National Council of Teachers of Mathematics and the Mathematical Association of America at Pittsburgh, Pa., Dec. 29, 1934.

[1] Snedden, David, *Secondary Schools in 1960*, Bureau of Publications, Teachers College, Columbia University, New York City, 1931.

[2] The *American Mathematical Monthly*, Vol. XLI, 1934, No. 3, p. 138.

education have afforded such an effective culture medium that the law of growth has seemed to be of an exponential type.

Meanwhile, the friends of mathematics have not been silent. They have pointed out numerous reasons why mathematics should continue to occupy a major place in the curriculum. They have suggested many ways in which mathematical education could be so improved that criticism would be less justified. A noteworthy example is the report on "The Reorganization of Mathematics in Secondary Education." To it we can trace much of the progress which has been made during the last decade. If the full spirit and program of that great forward-looking report could have become immediately effective, I believe that we would be in a much less precarious position today. To cite an instance, note that "correlation" and "integration" are educational shibboleths, and recall that the National Committee said: "There should be, moreover, a conscious effort through the selection of problems to correlate the work in mathematics with the other courses of the curriculum, especially in connection with courses in science."[3] In fact, thirty-two years ago this very day, in his address as the retiring president of the American Mathematical Society, E. H. Moore urged reorientation (and in particular, correlation with science) and invoked what he chose to call a vision of the future of elementary mathematics. He said in conclusion: "The successful execution of the reforms proposed would seem to be of fundamental importance to the development of mathematics in this country."[4] A generation later, we are still struggling to make effective some of the proposals of that address. When such disparity exists between our present achievement and the ideals visualized and enunciated by men like Moore, Young, and Hedrick, it seems to me that it would be futile to make at this time many proposals concerning either curricular problems or objectives. I have in mind a number of such proposals which I hope to make at some future time. Reorientation is useless, however, unless it is followed by action. The problems of reorganization of the curriculum and restatement of objectives have received a lion's share of attention in recent years. We have

[3] "The Reorganization of Mathematics in Secondary Education," reprinted (in part) by Houghton-Mifflin Company, Boston, p. 39.

[4] Science, N. S. Vol. XVII, pp. 401–416, March 13, 1903. Reprinted by the National Council of Teachers of Mathematics, First Yearbook, pp. 32–57 (out of print).

been orientated and reorientated, but too many of us loiter along the road. In my judgment, the most urgent need at present is concentration on the problem of achieving the goals now clearly placed before us.

Not long ago I asked one of the leaders of the progressive education movement what he would say in speaking on the need for reorientation of mathematics. He mentioned two things. First he said we must keep in mind the whole child, and referred to the tendency of subject-matter specialists to see things from a restricted point of view. Second, he said that we must show we are actually attaining the objectives we have set up. In his opinion the taxpayers of the next decade will be extremely critical and must be shown that they are getting their money's worth. If we assert that mathematics has certain cultural, disciplinary, and practical values, we must be prepared to demonstrate that pupils who have taken our courses are changed by them—that they are more cultured, better disciplined, and better prepared to solve the problems of real life.

If I read correctly between the lines, I see a problem which we soon have to face. Hitherto the attacks on mathematics have been largely confined to academic circles. The parents of our pupils have been staunch defenders of the fundamentals. The current economic problems have caused the taxpayers in many places to call for the elimination of the "frills in education." The so-called frills are designed to forward the development of the whole child, and in defending them I anticipate that the educators will in increasing measure carry the fight against us directly to the parents and taxpayers. We must, then, as part of our orientation prepare to defend ourselves *before the public* and take such action as will warrant and insure us a fair hearing in discussions of curricular reform. It is not enough to discuss these problems among ourselves, as we are now doing. In the past we have been too complacent in our conviction that mathematics is indispensable in the modern world. However distasteful it may be from a professional viewpoint, what we need now is some evangelism. In his address as retiring president, the late J. W. Young spoke on "Functions of the Mathematical Association of America"[5] and discussed at some length the popularizing of mathematics. I urge you to reread his

[5] See *American Mathematical Monthly*, Vol. XXXIX, 1932, No. 1, pp. 6–17.

remarks with the thought that such a program is not only desirable, but may soon become a necessity.

Unprejudiced educators agree that some mathematics is essential for a well-rounded development of educated citizens. Their attacks center upon the improper formalism of the traditional curriculum, the excessive rates of failure, and the evidence that achievement is of comparatively low order. College teachers of mathematics agree with them on the last point, at least. It is clear that to satisfy both groups we must first concentrate on better achievement by our pupils. Among the many suggestions for improvement which have been made, I shall mention only three—systematic diagnostic testing and reteaching, attention to individual differences, and maintenance programs.

Teachers should be so imbued with a mastery technique that it will be effective in the classroom. Such a technique has been most lucidly presented by Professor Morrison of Chicago in the formula: *teach*, *test*, *reteach*, *retest*, and so on, to the point of mastery.[6] The nearest approach to this formula made by many teachers is a superficial discussion of typical errors made when the examination papers are returned. They then pass on at once to the next topic where the process is repeated with accumulating difficulties and lack of mastery until many students are caught in a slough of despond and are bogged down in failure. Many good teachers resort to individual "make-up" work with extreme cases, but such attacks on poor achievement are relatively ineffective. What is needed is accurate diagnosis of particular difficulties during the learning process, followed by specific teaching designed to remove these special or individual difficulties *before* the achievement test is given. Such a program goes far toward preventing failures and producing a high order of achievement.

The question at once arises as to the method of grading the pupils when the mastery formula is used; that is, the problem of individual differences must be considered. Educators are vitally interested in this problem, but mathematicians have shown little interest in it and few attempts have been made to handle it on the collegiate level. In my judgment a solution which will prove satisfactory is based on the differentiated assignment or contract plan. Ability grouping may solve the problem for the larger schools, but

[6] Morrison, H. C., *The Practice of Teaching in the Secondary School*, Univ. of Chicago Press, 1931, p. 81 ff.

it is not feasible in thousands of smaller ones. At best it merely serves to divide the range of differences into intervals within which considerable variation remains. Some method is needed which is applicable within the individual class room. One good method consists in teaching the minimum essentials or "C-grade" work to the class as a group. As fast as diagnostic testing discloses sufficient mastery has been attained, students are encouraged to attempt successively higher levels of related work for grades of "B" and "A". Meanwhile, such reteaching as is necessary is given to those who need it. This means that students who show the most promise have the opportunity to work at more challenging and more stimulating content. They are the pupils who will probably make the greatest success in college, and the work on the upper levels can be definitely college preparatory. It seems to me that such a program may be adapted to almost any school. It puts emphasis on mastery and provides for the college preparatory work without disregarding the less gifted pupils. I look forward to the time when such procedures will be common practice not only in the secondary schools, but in colleges as well.

Although considerable progress has been made in providing for individual differences, there is much yet to be done. A committee of the Natural Council of Teachers of Mathematics has been working on the problem for several years. As a result of their work and the work of others, we should have much more definite information as to what pupils of different abilities can really master. Up to the present, we have relied largely on experience. Because of the scope of their problem the committee has confined its attention to the slower pupils. From the collegiate viewpoint, it is desirable that attention be directed toward the more gifted, and the committee will doubtless in due time so extend the range of its investigation.

In addition to methods which point toward mastery and provide for individual differences, a third program is necessary if we are to satisfy both educator and mathematician. I refer to the need for carefully worked out plans for aiding the retention of things once learned. Some of the better schools now have such maintenance programs but they are the exception rather than the rule. Certain schools have reduced instruction in algebra and geometry to four or even three days a week. I would not oppose such reductions if all pupils continued the study of mathematics in the succeeding

years for one or two days a week. Such a plan would permit an effective maintenance program and allow time for maturation. The attainment of mathematical maturity takes time. There have also been proposals that algebra and geometry be moved to the last two years of the secondary school. It seems to me that the first plan has many advantages not shared by the second.

In this connection I would like to consider briefly a point connected with the articulation of secondary school and college. College teachers, high school teachers of science, and others who expect pupils to know high school mathematics, occasionally complain about certain gaps in such training which they discover. In many cases the fault may be with the college teacher or science teacher. He may be ignorant of modern courses of study and be looking for obsolescent material. To cite an instance, suppose he discovers that his students do not know how to handle proportions. The reason may be that he writes his proportions in the form $a:b::x:y$, and if such is the case, the fault is clearly with the teacher. Following the recommendation of the National Committee, the use of the double colon in proportion is practically obsolete in modern high school books. E. H. Moore said that college teachers need to keep in closer touch with secondary school mathematics. This is a recommendation for personal reorientation which many have failed to follow.

The converse of the above situation may, of course, hold; namely that the secondary teacher has taught the obsolescent form and the fractional equation used by the college instructor is not recognized by the student as a proportion. A properly constructed diagnostic test by the college instructor might reveal that a given pupil really knows something about proportion after all, if it is presented in a form familiar to him. For a comprehensive discussion of this subject I refer you to two articles by Professor E. R. Breslich in the October, 1933 issue of THE MATHEMATICS TEACHER,[7] and the February, 1934 issue of *School Science and Mathematics*.[8]

Many educationists attack mathematics because of their belief that it is overburdened with formalism. Three primary elements compose the class-room situation—the pupils, the textbook, and the teacher. Experienced teachers who have consciously tried to avoid improper formalism know that many pupils seem to like it.

[7] THE MATHEMATICS TEACHER, Vol. XXVI, No. 6, pp. 327–49.

[8] *School Science and Mathematics*, Vol. XXXIV, No. 292, pp. 144–57.

The period never runs more smoothly than when the pupils are engaged in some formal exercise or manipulation which requires little real thinking. It is far more difficult to teach them to think than to respond formally. To use a term which Felix Klein attributes to Thomoe, "thoughtless thinkers" are the majority, not the exception, in the high school population of today. This tendency to mental inertia is a strong influence toward improper formalism, but it is not one which should be blamed upon mathematics or mathematics teachers. It has frequently been pointed out that because of the objective nature of mathematics such traits are readily observed in mathematics classes. It is hard to believe that the quality of thinking improves when pupils pass down the hall to a social studies class. It may not be as easy, however, to detect that they are substituting a mass of verbiage for the concise exposition of an idea.

The second element in the classroom situation is the textbook, and if often serves as a guide on excursions into improper formalism. Many of our critics, however, seem to be unaware that great improvements have been made in choice of content, in arrangement, and in presentation. Improvement is slow, but nevertheless recent books are far superior to the old ones.

The third primary element in the class-room situation is the teacher. Overworked and undertrained, the average teacher follows the text closely and gravitates toward improper formalism. The poor preparation of the average teacher is so well known and so often remarked that I shall not enlarge upon the subject here. The MATHEMATICS TEACHER of December, 1932, contains an article by Professor E. R. Hedrick on "Formalism in Mathematical Teaching."[9] In this paper he discusses the training of teachers and makes clear the distinction between what is proper formalism and what is improper. He also discusses some of the other influences which promote it. Among these are certain types of tests. The examinations of the College Entrance Board and statewide testing programs have often been criticised for overemphasis on formalism. A commission is now studying the problem of the College Board examinations. We have every reason to hope that its recommendations for future practice will tend to allay the criticisms of the past. Much criticism of testing programs is just, but such programs

[9] Hedrick, E. R., *Formalism in Mathematical Teaching*, THE MATHEMATICS TEACHER, Vol. XXV, No. 8, p. 449.

are not wholly bad. One illustration will suffice. Two years ago in a certain middlewestern town, the school board as an economy measure discharged its mathematics teacher. The classes in algebra and geometry were put in the hands of a teacher wholly untrained for teaching mathematics. This is a practice which has been denounced for years with little effect on school boards and principals. This state, however, has a testing program, and when the returns from the tests came in, the principal by means of objective evidence was able to convince his board that what they needed as a good mathematics teacher. The point I want to make is this: To theoretical haranguing on the superiority of the trained over the untrained teacher that board would doubtless have nodded assent and done nothing. However objectionable those tests may have been from some points of view, they provided clear-cut evidence and produced immediate results from that school board and principal. A testing program properly conceived and carried out may be one of the most powerful forces tending to produce improvement. It seems to me that we should direct our energies toward seeing that the program is so designed as to bring about a maximum of improvement with a minimum of the objectionable features which invite attack.

When all of the tendencies toward formalism—the pupil, the textbook, the poorly trained teacher, an improper concept of tests, and others, combine and operate together as they often do, in a single class-room, the results are indeed deplorable. The road to improvement is through the textbook, for it serves as a guide for pupil, teacher, and testmaker. I have already referred to the improvements which have been made. The greatest room for further improvement, it seem to me, lies in the direction of expanded discussions of the underlying principles. For some years the conviction has been growing within me that this is an essential step for further progress. Such discussions are now beginning to be more common, particularly in geometry, where the need is most acute.

There are those who argue that expanded discussions are out-of-place in textbooks—that the author is usurping the prerogatives of the teacher. It seems to me that the reply to this is that the majority of teachers has been, and for some years to come, will be, unprepared to do the job well. The author is far better prepared to do it than most teachers, and if it is not done by the author, it is often not done at all. Moreover, if a pupil fails to get the point

of an oral discussion, or has been absent from class, he need not be retaught by the teacher or else lose out entirely. If a discussion is printed in the textbook, the pupil has an opportunity to study and restudy it as needed. A well-written discussion is of great value to the poorly trained teacher, and such a discussion need be no handicap to the well trained teacher. If opportunity arises, there is much more to be said on the principles, purposes, and methods of mathematics than an author can put within the covers of an elementary textbook.

The above remarks lead directly into the subject of transfer of training. As research continues, evidence is accumulating to the effect that many current theories are not as sound as their supporters would have us believe. Early experimental results seemed to indicate that transfer was limited, and this gave rise to the specific-habit psychology. In this field Thorndike has been a leader. Recently the neurological experiments of Lashley and others, and the experiments of the Gestalt psychologists, have thrown some doubt upon the validity of Thorndike's theories. In an article published in the *Psychological Review* during 1933, three leading Gestalt psychologists discuss "Inconsistencies in Thorndike's System"[10] and assert: "In recent years, however, Thorndike has added much to his system. It is the purpose of this discussion to show that without exception each advance substantiates Gestalt psychology." Moreover, in his retiring address as president of the American Psychological Association, K. S. Lashley referred implicitly to Thorndike's theory when he said that recent experimental and clinical evidence seems to present fatal difficulties to certain theories.[11] In another place, he writes, "We cannot, on the basis of our present knowledge of the nervous system, set any limit to the kinds or amount of transfer possible or to the sort of relations which may be directly recognized."[12]

Time does not permit an extended discussion of these matters. The point I wish to make is that the psychologists are unable as yet to agree among themselves. What seems to me to be vital is that many educators, apparently accepting as *fact* what is in reality only *theory*, have gone about the reconstruction of the curriculum

[10] *Psychological Review*, Vol. 40, No. 4, July 1933, pp. 303–323.

[11] *Psychological Review*, Vol. 37, No. 1, Jan. 1930, pp. 1–24.

[12] Lashley, K. S., *Brain Mechanisms and Intelligence*, Univ., of Chicago Press, 1929, p. 173.

rather precipitantly. Many apparently believe that learning is specific habit formation, that there is little transfer of training, and that only the mathematics which pupils may reasonably be expected to need should be taught. They overlook or ignore the fact that mathematics, if properly taught, is peculiarly adapted as a vehicle for concept formation—particularly the principles of induction and of deduction and the laws of sound reasoning. These are in addition to the *practical values* which hinge upon factual information and particular skills, such as the ability to use a formula, or interpret a graph, or solve an equation.

There is general agreement that transfer is not automatic, and that we must train for transfer. We must do this by continuously keeping our pupils conscious of the generality of the methods we are using. In algebra through emphasis upon inductive methods they can be taught to discover and to generalize, and can be led to see that day after day they are using methods through which civilization has made many of its great advances. In geometry, without neglecting the inductive discovery of theorems, the emphasis may be placed upon the principles of deductive proof. The critical thing, however, is that through repeated discussion they must be made conscious of these underlying principles and of their tremendous generality. Mathematics is the world's finest exhibit of the application of these principles, and because of the simplicity of its elementary concepts and postulates, is a superior medium for the exposition of them to developing minds. Entirely apart from the practical values of mathematics as a tool subject, I think it is safe to say that through acquisition of these ideas our subject may possess tremendous cultural and social values for the student.

The University of Wisconsin in its Senior High School is now conducting what is probably the most radical curriculum experiment yet attempted in this country on the secondary level. Taking as objectives the categories of health, worthy use of leisure, community living, and vocation, a selected group of students is attacking these problems directly. This is a definite attempt at functional or specific education for adjustment to the problems of everyday living. But for me the most significant feature of the Wisconsin High School program is the explicit recognition that the cultivation of intellectual capacity does not necessarily call for a different type of curriculum. For this purpose modernized courses in mathematics based on the traditional patterns are not

unsuited. If more educators would make a distinction between education for adjustment and education for intellectual capacity, I feel confident that much of the criticism of mathematics, based as it is on the social utility theory, would disappear. Educators are stressing the adjustment program, while mathematicians retain their allegiance to intellectual development. I see no reason why we cannot have both. This will, however, involve closer co-operation and better understanding between the opposing forces than exists at present. This personal reorientation should be a major objective for all of us in the immediate future.

In conclusion, I wish to reiterate my belief that curriculum reorganization alone will not relieve us from the sting of criticism. Better achievement through improved classroom methods is a necessary component of the forces which will carry us forward and stem the ebbing tide of mathematical instruction in the secondary schools.

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The 1002nd Arabian Night

By BLANCHE B. HEDGES
Ogontz School, Pennsylvania

THE PALACE was aglow with the light of many candles. Though Scheherazade II had finished her 1001st story, she promised many more, so Al-Mamun was a happy man.

And why not? Was not his capital the most brilliant in all Asia? Was it not splendid with beautiful buildings, and "rich beyond the dreams of avarice"? Was it not the center of a vast trade between the Orient and the Occident? Yea, verily! Its streets were alive with busy peoples and endless caravans of camels from the desert of Gobi, elephants from India, and prancing, snorting, sensitive steeds from Araby! But best of all, had not *he*, Al-Mamun, made it also the intellectual center of his time? *He* had attracted to it scholars of all nations, Hindu astronomers, Greek philosophers, Jewish and Arab doctors, Al-Kindi, Al-Fazari, and last and best, Mohammed ibu Musa Al-Khwarizmi, now the royal astronomer. Indeed this 1002nd night was a celebration, because the latter had finished his great book, "Al-Gebr we'l muckabala."

Al-Mamun ended his revery with a deep sigh of content, and lay aside his still smoking narghileh to take a tiny cup of coffee essence from the beautiful hand of Scheherazade, who reclined on the divan beside him.

"Proceed, Mohammed, son of Moses! I would know more of thine early life."

"Oh Commander of the Faithful," replied Al-Khwarizmi, proudly as he made a low obeisance, "Thou who knowest all things will recall that I come from Khorassan below the Caspian. . . ."

Just then a loud tumult arose in the courtyard and cries were heard:—"Away with him! Off with his head!"

The Most Exalted One with a wave of his hand dismissed the Grand Vizier to investigate. In a few minutes three powerful mamelukes entered the room dragging a man after them.

"Oh Sire! May the Prophet bless thee!" began the Chief Eunuch. "While we were praising the new book of the numerals by Al-Khwarizmi—may fame enrich him for all time—this stranger asked

certain questions of us, then said "Pooh-Pooh! Those numerals are no better than many others I have seen."

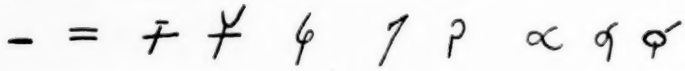
"What! Al-Khwarizmi, All Knowing One! Other better numerals than the ones we have adopted? Thou saidst this kind was the best ever, the best in market place and in bazaar. Explain! I brook not being tricked. Remember my everlasting reputation as a patron of the arts and sciences is at stake."

"True, Oh Commander of the Faithful! These numerals we are making immortal through our adoption are the best the world has yet seen. These in my new book are of Hindasi origin, I believe, though there is no authentic literature on the subject. However interviews with the Hindu merchants who throng our bazaars confirm me in my opinion.

"Al-Hindu, my predecessor in thy royal chair of astronomy in '73, through his masterly astronomical tables first brought my attention to these symbols. I then began a study of the works of Arya-Bhata, Brahmagupta, Mahavira—may undying fame attend them! Internal evidence leads me to believe they knew and used these numerals in their laboratories, not deeming them sufficiently scholarly, being of the people, to be put in their most learned works. Possibly that is why they were lost!

"Now Sire, there is one chapter in my life that I have kept secret unto this day, but which I now reveal. In my youth I myself journeyed through India. I studied the ancient Vedas and the Brahmasutras. I spent many hours over the inscriptions of King Asoka, and those in the ancient caves at Nana Ghât and at Nasik. I found characters like these."

He unwound a prayer stole from around his neck, and pointed to these number symbols embroidered therein.



"Surely these symbols, Sire, beat the strange systems of other peoples, who use words, initial letters of number words, or the alphabet—very confusing, methinks!"

"Stranger! Thou hast heard my royal astronomer," said the Grand Caliph. "I am content! Vizier! Off with the stranger's head!"

"Oh Prince of the Faithful! Let me tell my tale: then judge between Al-Khwarizmi and me." Al-Mamun bowed his consent.

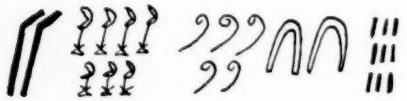
"First, know thee, that I am called Sindbad the Sailor. When I was born, the thumb on my left hand was folded against the palm, while that on my right was holding down the little finger. All the other digits were stretched out straight. The genii deduced therefrom that I was the son of the Prophet, and would lead a charmed life: for in my hands I held the perfect 3, the evenly even 4, which together compose the mystic 7. The genii said I would live a long life in 7 cycles, and that great things would happen to me. My boyhood and my youthful days were uneventful, but

Cycle I

"One day in my young manhood I felt that something queer had happened: I did not feel a bit like myself. I looked down at my dress and found it strange—a long white robe—while on my head was a golden bandeau. Upon my shoulder perched an eagle, which obeyed my slightest word. I gazed around me: I seemed to be standing on the sands of a great desert, and I descried in the distance huge forms of stone, which on approach proved to be pyramids, those of Egypt I doubted not. I concluded from my robe and bandeau that I was a philosopher, and set out at once to improve my opportunities for study. Proceeding to the city I presented my card to the High Priest, who initiated me into his cult, "giving me directions for knowing all dark things." This was an unparalleled opportunity and I studied everything in that country, their customs, their religion, their arts and sciences. In passing I must mention one thing which particularly piqued my curiosity, over the proof of which I have pondered much. It was the method* used by the engineers or harpidonopti for the orientation of the temples. But that is another story—I must not digress.

"Now, it seemed to me, that the crown of the civilization of the Egyptians was their ability to make captive the spoken word, which so soon as is, is not! They had three methods of writing, and each had its own system of numbers: the hieroglyphic, the hieratic or priestly form, and the demotic or vulgar forms. In the demotic system, by the repetition of these number forms, and by the principle of periodicity they could get any number. Their idea of position was a crude one, the largest denomination at the left, and the smallest at the right. Thus 27,529 was written like this":—

* Method for making a right angle by the 3-4-5 triangle.



"All went well along the Nile for many fruitful years, when suddenly

Cycle

"I found myself at the head of a large caravan journeying by easy stages along the Tigris. I was a rich trader, I discovered, carrying in my camels' packs silks, rich velvets, dates, musk, attar of roses, and salt to Babylon, the City of the Great King. And that was some city, a hive of commerce, with beautiful buildings, and carved statuary, and wondrous hanging gardens. But in the revelry and later in the terrible excitement and confusion of that fateful night when the writing appeared on the wall, I completely lost my head. I, too, seemed to have been 'weighed in the balance and found wanting,' for when I came to, all my wealth was gone. All I had to repay me for my travels and my work, all I could even remember of the Babylonians was their number system."

"Now it happened that in that hot country there were no papyrus swamps such as the Egyptians had, so the Babylonians made tablets of soft mud, which they pricked with a wedge-shaped stylus, then baked in the sun. Thus thou see'st the character of their writing was determined by the nature of the materials and instruments used. I have seen literally thousands of these tablets containing bills, accounts, number processes."

Cycle

"Oh Prince! Live forever! Imagine my interest when, in my third metamorphosis, I found myself near the homeland of the Lost Tribes of Israel, whom I had met in Babylon. You see I had sailed along the coasts of Phoenicia and landed at Tyre after a stormy voyage, probably from Egypt. The Phoenicians were sailor-traders, and on the docks of their seaports I found merchandise marked for all parts of the known world. I could not read these marks, but to my mind they showed an Egyptian influence with hieratic tendencies.

"Later I journeyed into the interior over beyond Sodom and Gomorrah to the holy city of the Hebrews, Jerusalem. Here I found the situation was sadder and more tragic than I had dreamed for it was 10 tribes that had been carried off into the Great Captivity. Now the more I pondered that Hebrew character for 10, the more curious it seemed to me, a mere dot, for the symbol of the perfect triangle. I, therefore, employed a scribe to tutor me in that branch of learning. Remembering that the only thing I brought out of Babylon was the Babylonian characters, I felt I ought to be a little forehanded.

"The scribe proved to be a poor teacher: he was a Pharisee at heart. But he showed me this much. These are the numbers 1–10, and thou must read the row from the right to the left as in all Semitic languages."

"This was all I could take in, however, and I became weary and fell asleep."

Cycle Λ

"When I awoke I looked up into the brightest, clearest, bluest sky I ever saw. I leaped to my feet with an agility I had never felt before. The soft music of the lyre was coming from a beautiful green grove nearby, so I strolled thither. There I found a group of earnest men listening to one who, as he talked, walked back and forth in the portico of a lovely summer house. I drew near to listen as I was only travelling for my health and amusement, and had all the time in the world. His talk was interesting, a scientific exposition of something he called arithmetica. This seemed to be a philosophic discussion of number. However, I caught some condescending references to a logistica which from the context I assumed to be the practical use of numbers in calculation. When I went up to the city later, there were many who were willing to talk it over with me, 'for all the Athenians and strangers who were there spent their time in nothing else but either to tell or to hear some new thing.'

"Their system seemed a bit confusing to me, because they used their word-elements to represent number, differentiating between

the letter and the number by a small mark. I have since learned that they improved upon this form at a later date using for the first twenty-four numbers the Ionic alphabet in the fixed order of custom.

"The invigorating climate of this lovely land and contact with keen, bright minds spurred me on to great activity, and I began a series of researches into the origin of their number system. I learned that in antiquity, as far back as the time of Solon, the Law Giver, the Greeks designated their numbers by the capital initial letter of the name of the number. For instance:—π stood for pente (5), Δ for deka (10) and H for 'ekaton (100).

"But 'ekaton seems to begin with epsilon!" said Al-Mamun, as he contentedly smoked the pipe that had not yet been invented.

Sinbad ignoring the pointed criticism of Al-Mamun continued his narrative. "These researches opened up a new line of thought to me. I began to see that the present can be studied and understood only in the light of the past, with an eye on the future. This tremendous and fruitful idea I carried over with me into my next experience, thereby enriching it to a great degree."

Cycle V

"Bang! The sound of the catapulta restored my dazed senses. I, legionary soldier of Rome, backed by the SPQR, Senatus Populusque Romanorum, veteranus of many campaigns, had been momentarily staggered by the stubborn resistance of the Belgae.

"Horum omnium fortissimi sunt Belgae.

"I had been with Caesar Imperator in all his campaigns in the West, serving in the cohort of the quaestor. Here in the business necessary to the proper provisioning and care of the army, a thing close to great Caesar's heart, I had come in contact with the practical side of military methods. Long years of struggles with the impedimenta and the distribution of stores to the equites and to those hungry hoplites had taught me the necessity for recordable numbers and I had become expert in their use.

"Whenever I lecture on the subject of Roman numerals, I am so full of enthusiasm for my subject, and there is so much material, it is difficult for me to give a unit course. However since my very life depends upon it, I shall try.

"A study of their numerals shows that the Romans understood

the subtractive principle, and denoted it by placing the subtrahend before the minuend. They also knew the addition principle, showing that operation by putting the smaller number after the larger.

"We must note here and bear in mind that the Roman Empire was a vast and mobile thing. As a consequence the numeral system was ever in a fluid state, so it is easy to see how various numeral-dialects could arise and yet be good form.

"It was always of interest to me to watch the reaction of the auxilia and the native camp followers to our Roman system. Belgae fortissimi sunt, but Galli and Acquitani had the keenest number sense, had it indeed to a much higher degree than the slower Anglians whose little island we invaded at another time. A word to the former in explanation seemed to suffice, as it does to thee also, Oh Commander of the Faithful. Indeed the Acquitani were very fruitful ground and received my instruction with an avidity most flattering to the subject. This led me to offer one year when in castra hiberna, a post gradum course in 'The Comparative Study of Modern Roman Numerals with those of the Aboriginal Etruscans'. It was a very popular course.

"And now, Sire, I have finished my narrative—that of 5 cycles. I rest my case! I believe the future holds the remaining two. 'The fate of every man is hung around his neck!' Let Al-Khwarizmi now support his position."

And Sinbad the Sailor kissed the ground.

"Yea! Or off with *his* head!" said Scheherazade and Al-Mamun together.

Mohammed ibu Musa Al-Khwarizmi the Thoughtful rose to his full height.

"Oh most Faithful Commander of the Faithful! Live forever! Grant me first to cast thy most august horoscope! "—hoping to gain a little time.

Al-Mamun gave his most gracious consent. The Court waited. At length Al returned, trembling with a visible but suppressed excitement.

"Oh most Noble Patron! Great beyond my dream! It is I, thy humble servant who will make thee live forever!"

Thereupon he waved aloft a parchment scroll, on which were many mystic marks. These mystic marks were the symbols for 1–10, as they have come down through the ages, developing from

the strange characters of the Brahmi, and of Boethius, to the Gobar, to the Eastern Arabian, and finally to the modern forms.

"Behold! These names on the left are the names of these periods, or of the men who led them, and the signs following are the number symbols in the changes of their evolution.

"Dost thou see the most wonderful sign of all—the one for which the world waited long? It is the closed circle around which the other numbers move, as the stars around the moon, falling into place only through the agency of its nothingness, paradoxical as that may sound. Thus 43, 403, 430 have different values because of the presence or absence of that little sign of the void."

Completely absorbed in his contemplation of the chart Al-Khwarizmi continued.

"I see the content of my book, 'Al-gebr we'l muckabala', composed under thine inspiring patronage, carried all over the known world by other Sindbads. I see scholars and merchants and the needs of the Times working over the progress of the race, rearranging, adapting, improving, passing on, as I have tried to do. I seem to see number sense and civilization growing and advancing together—each the function of the other. It is all down in this parchment.

"And thou, Sindbad! I see thy two remaining journeys. As Gerbert of Acquitaine thou wilt be the advance agent in the West for the Gobar numerals, with main office at Rome. Through thee they will be introduced into Europe.

"Lastly I see thee‡ coming up from the south—a tiny, flickering flame like that of a bougie in thy hand, the faint light of which is destined to be a Light to Europe!"

During Al-Khwarizmi's long speech all eyes had been focussed on the mystic scroll. He beckoned to a mameluke who took it from his hand and hung it on the wall. Now a strange thing happened. In a moment the figures were gone, the slate as it were, was wiped clean. Fascinated, or awed, or fearful, or eager the Court continued to watch. In a series of dissolving views rows of number symbols appeared one row at a time, each on fading away being replaced by the next. Thus before their waiting, watching eyes passed the panorama of the Middle Ages, the laborious, simplifying process under the sometimes clumsy fingers of cloistered monks. Last of all appeared the date

‡ Fibonacci of Bugia.

1478

Al-Khwarizmi gave a shout of joy.

"Allah be praised! I see my name preserved in Algoritmi and in Augrime. And I see our much belabored numerals fixed in the first Arithmetic Textbook to be printed—at Treviso in 1478.

"Sire, I see no more. But spare Sindbad,* I pray thee. He is of those who in each age do their part in the lifting of the veil that hangs between Mankind and Revelation. Spare him. The Future claims him."

Al-Khwarizmi bowed his noble head in generous supplication for another's weal. His eye once more caught sight of the scroll. He gave a start.

"Sinbad! Didst thou say that 7 cycles were ordained to thee? This is passing strange! I see an 8 intertwined with thine aura. What means this? Hold! Ha! I have it." Then turning to Scheherazade,—

"Oh Scheherazade! Glorious moon Princess! I see many groups of little moon maidens beautiful as thyself, writing upon black walls with pure white stones. I see them using the numerals of our adoption. I hear one ask: 'Where did these pesky marks come from, Teacher? Who made them?'

"In answer *Sindbad the Sailor* comes forth from the mists of the Past. He brings a wealth of experience to a great task. He gathers up the threads of dim antiquity, following this clue and that trail hither and yon over the face of the globe, bringing always a fine sympathy, a living imagination, and a true sense of values to the solution of the problem of the little moon maiden. This time his wealth does not fade away into nothingness as it did by the waters of Babylon, for he has caught the last word in a Great Book—imperishable and a help forever!"

In silence the Court turned as one man to pay honor to the two defendants. Then Al-Mamun spoke.

"Grand Vizier! Open up the royal coffers! Bring forth silver and gold, and pearls and diamonds! Shower them upon Mohammed ibu Musa Al-Khwarizmi—Man of Vision! And upon Sindbad—Citizen

* Doctor David Eugene Smith
Emeritus Professor of Mathematics
Teachers College, Columbia University.

of the World! Scholar! Philosopher! Historian! Teacher! Friend!"

"Let them live in the Heart of the World forever!"

Such was the royal decree of Al-Mamun, Grand Caliph of Bagdad, in the year of the Hegira 208, and of Grace 831.

Allah be praised!

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See the final program of the Annual Meeting of the National Council of Teachers of Mathematics at Atlantic City on page 121 of this issue.

Vitalizing Geometry Through Illustrative Material*

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MOST high school teachers are likely to feel that college professors are theorists whose ideas would be more practical if they were actually engaged in teaching the high school pupil of today and if they were actually confronted by the problems which most of us are forced to meet. We have heard much discussion about educating the young people to take their places in a new social order, about educating for leisure time, and about the necessity of vocational guidance. Yet most of us have not been serious enough about it to realize that we should have a new philosophy of teaching and should alter our courses of study to meet new objectives.

In spite of the fact that we as mathematics teachers should have been among the first to change our point of view, most of us have continued in the same old paths concerning ourselves with problems of method and class room procedures rather than with the question of how the main objectives of a high school education should affect the content of our particular courses. And while we have been asleep we find that the percentage of high school pupils taking geometry decreased from thirty per cent in 1910 to nineteen per cent in 1928. The committee on the "Relation of School and College," appointed by the *Progressive Education Association*, has twenty-seven secondary schools at work remaking their high school curricula. In these twenty-seven schools which are free to work out content and method as they choose, mathematics is greatly reduced. After a minimum requirement ending before or after the completion of elementary algebra, mathematics work becomes elective, and the chief emphasis is to be placed on social science. This is but one example of the general attitude that mathematics does not have an important part to play in either the cultural or industrial life of today.

Our general educators tell us that the purpose of education is

* An Address given before the Mathematics Section of the Schoolmasters' Club Meeting in Ann Arbor, April 27, 1934.

"preparation for life," or as one has expressed it, "to make the world a better place in which to live and a better place in which to make a living." I contend that although we may be very much at fault at the present, our mathematics courses and especially geometry can contribute to the life of every high school pupil whether he be the dull or the bright individual. Failure to pass indicated originally that the pupil was not adapted to the course and the pupil was dropped; failure is now interpreted as indicating that the course is not adapted to the pupil and the course is dropped or modified. At present the provision for students of lower mentality is inadequate and unfair. Equally unfair is the inadequate provision for students of superior ability. So can we not recognize and preserve the good in both the new and old philosophies and devise means to serve better both the old and new types of students, neither of whom at present receives his due? So, realizing that some of our teaching has been bad because the product is bad, let us sort out some of the basic values.

There is practical agreement that the main justification of demonstrative geometry relates to logical thinking, at least so far as this is confined to mathematical thinking. According to the *Gestalt* psychology, if we wish to understand an idea we cannot do so until we see it in its proper relationship to the culture of which it is a part. It is the study of ideas or things in relationship which gives meanings. Hence, we need to bring into our class room applications from everyday life if the pupil is to have a better understanding of the theorems which have been studied from the time of Euclid to the present day—in other words, bring in illustrative material in order to give more meaning to what we have always studied.

But for reasons even more important than this, our geometry needs to be vitalized. We are gradually recognizing the fact that leisure, even more than work, is likely to be the chief occupation of man's time in the future and that every school needs to make provision for hobbies in fields which will keep the individual physically and emotionally sound. In some schools mathematics clubs can accomplish this purpose for the student who is already interested. But those who have no liking for mathematics and who most need help in developing some interest will not join such a club and so need material brought to them in the class room.

If the pupil is to gain any benefit from geometry or any other

high school course, he must be convinced that the material of instruction is worth while. It is possible that some of the pupil's dullness may be a rather intelligent decision on his part not to do things which he has decided are of no value to him. If we can open up a field of possibilities for making mathematics vital to the student by appealing to his interest in things which he meets daily, we can accomplish many things for both the dull and the bright, the interested and the disinterested all at the same time.

If we can bring material into the class room which will make the pupil realize that a skyscraper, an automobile, a bridge and a tunnel represent applied geometry; that hundreds of trades depend for their very existence upon precise measurement, on blue prints and scale drawings; that the contractor, the builder, the mason and carpenter must appeal to geometry to avoid waste of material; that by geometry the surveyor makes his map, the civil engineer his railways and the sailor navigates the ocean with success; that the geometric principles of equality, symmetry, congruence and similarity are implanted in the very nature of things; that many of the fine arts such as designing, painting, etching, sculpture, making of pottery and jewelry and the use of art in advertising, all have a geometric foundation; that numerous industries such as those concerned with the making of clothing, furniture, textiles, linoleum and wall-paper, would all be crippled without a knowledge of the geometric methods employed in drawing; if we could in some way bring all this into the class room, then we could accomplish many purposes at once. We could better motivate the traditional subject matter; we could give the pupil an opportunity to gain an interest which he might develop either for leisure time or for an occupation; and we would give an opportunity for vocational guidance by making the work sufficiently flexible. We would increase the culture of the individual by making him better realize the part geometry has and is playing in our civilization. A topic of practical value to one pupil has the possibility of great cultural value to another.

It is with these things in mind that I have organized a unit in *Plane Geometry* which I wish to describe to you. I call this unit "Geometry in Nature, Art and Industry"; however, the traditional subject matter covered is what is commonly known as Book Five on "Regular Polygons and Circles" and includes a study of *point and line symmetry*. All through this unit the emphasis is to be

placed on practical applications. Instead of an entirely theoretical study of the topic we shall continually refer to the uses of geometry in nature, art and industry as illustrated by any materials which we can gather. And while our emphasis will be on uses of regular polygons and circles, the student will be encouraged to find and bring to the class room practical uses of any principles of geometry which he may see illustrated.

The teacher will have prepared a bibliography for the pupil which he may use as he wishes. The last three days spent on the unit will be devoted entirely to the practical applications and to the exploration of certain vocations in which a knowledge of the forms and principles of geometry is required. The class will divide itself into groups according to interests such as engineering, astronomy, architecture, bridge-building, and surveying, and the pupils will make their own assignments for these three days. For example, they may decide that one day will be "engineering day" and another day "nature day." During these days the reports will be voluntary and the discussions socialized. But inasmuch as motivation does not and should not eliminate work, each pupil will be required to make one rather complicated original design which might be used for linoleum, wall-paper, stained-glass window, landscaped gardens or anything in which he is most interested. Use will be made first of any illustrative materials which the pupils will bring and the teacher's material used for supplementary purposes later.

I will describe to you some of the illustrative material which I have hurriedly collected for myself; however, you probably could make a much better collection and a really complete one can be made only by gathering it little by little as one finds it over a period of years.

Nature furnishes its share of geometric designs. Among the flowers the black-eyed susan, daffodils, dahlias, trailing-arbutus, and the water-lily are excellent illustrations of point symmetry while the leaf butterfly furnishes an example of line symmetry. The bird builds a symmetrical nest, the honey-bee builds hexagonal cells and the starfish has its pentagonal form. The spider makes a web consisting of all sorts of polygons. Snow-crystals represent beautiful hexagonal formations. Landscaped gardens are often symmetrical and laid out in regular polygonal designs. In fact,

much of nature is symmetrical as is evidenced by both plant and animal life.

Illustrations from industry are almost endless. Comparisons and contrasts in bridges built in different countries in different stages of civilization will interest most any boy or girl; for example, the George Washington bridge crossing the Hudson at New York City, the Arlington-Memorial bridge, the Victoria bridge near Windsor, England, and Madrid's Toledo bridge built in the eighteenth century. And in contrast to all these is "Devil's bridge" in Spain which was built eighteen centuries ago with no mortar, no cement and no rivets holding the granite blocks in place. You are probably wondering of what value is this when but a small percentage of our pupils will be bridge builders. I can enjoy looking at a beautiful picture but because of my lack of knowledge of art I cannot appreciate it as many folks do. Likewise, most folks ride on a bridge and all they know is that it is smooth riding; but if they can appreciate, even in a small measure, some of the principles which have been used in making that bridge, then life is just that much richer for them.

Surveying instruments with brief descriptions of their uses may interest some pupils. The angle-mirror, hypsometer, plane table and sextant could not accomplish their purposes without the use of principles of geometry.

House-furnishings such as book-covers, clocks, and lamps are decorated in geometric design. The boy who hates geometry but who enjoys working in the woodshop may enjoy looking at the designs which may be found on American, English, French, and Italian furniture. About the most interesting illustration of furniture which I have is the picture of an Egyptian desk made in the fourteenth century.

Most boys will enioy finding examples among machines. The automobile, the locomotive and the ship illustrate automotive machinery; McCormick-Deering tractors and pulverizers illustrate agricultural machinery; commercial machinery is illustrated by the typewriter and industrial machinery by the Baker Valve Gear. Students who are taking work in the machine shop may be able to contribute interesting discussions of the principles of geometry which they use in their shop work.

Girls who hate mathematics may enjoy looking for the use of geometry in the making of textiles. Parallel lines, triangles, rec-

tangles, trapezoids, circles, sectors and all sorts of combinations are used in the designs. The study of congruence and similarity will be much more interesting when it is discovered that in a piece of material the figures in the pattern must be congruent or similar to make a material attractive.

A study of geometry in art is an example of how supplementary material may be used in handling exceptional cases. Consider the girl who is specializing in art. She is an average pupil in all her classes except in art and mathematics; in the former she is exceptionally brilliant; in the latter she has great difficulty, she is not interested; she does not apply herself and does not care whether she learns or not. Her interest in mathematics may be aroused through art. She is willing to work on a project whether in mathematics or not, provided she can draw. A supplementary project based on harmony of line or color or one based on symmetry may give this pupil a new viewpoint of what mathematics really is. And she may be able to contribute much to the class concerning the use of geometry in art.

Regular polygons and circles are used in commercial art such as in advertisements, in graphs and in making conventional designs used in blankets, lace, rugs, tile, and wall-paper. Embroidery, quilting and weaving illustrate its use in domestic art. And the fine arts make constant use of similar designs. Crystal tumblers, stained-glass windows, Egyptian paintings and Mexican, Greek, and Chinese pottery are interesting illustrations.

The study of architecture will appeal to almost any boy or girl. Some may have attended the *Century of Progress Exposition* and will have pictures showing the exterior of the buildings. They may enjoy looking at pictures of other modern buildings such as the Chrysler Building and the Waldorf-Astoria Hotel. Interesting comparisons can be made of both the interiors and the exteriors of the early American home and the modern American home. The architecture of bridges I have already mentioned. It is fascinating to compare the simplicity of the early American church such as the First Church of Lenox, Mass., with the elaborate cathedrals such as the Cathedral of Notre-Dame, Rheims Cathedral and the Cathedral of St. John the Divine. Doorways, mausoleums, and museums furnish examples of other types of architecture.

While I have classified these illustrations under nature, art or industry, many of them illustrate more than one thing, especially

those listed under art and industry. Boys who are interested in aviation will enjoy looking at a chart devised by a committee of mathematical experts for checking Commander Byrd's polar flight data. A chart showing the route Lindbergh followed in his trans-Atlantic flight is a good illustration of the location of aerial routes along great circles. A diagram showing the parcel-post zones of the United States is an example of the use of circles. Boys who take Architectural or Mechanical Drawing will have drawings and blue prints to illustrate the use of similar figures. The pupil who dislikes mathematics but likes athletics may enjoy making diagrams and explaining to the class how geometry is used in laying off the basket-ball floor and the football and hockey fields.

The bibliography which the student might use would depend of course upon the library facilities of each particular school. Trade catalogues, books on applied arts, periodicals and advertisements, will assist in transforming the geometry of the class room into a livable reality. I will mention but a few readings which I think are both enjoyable and valuable to the student. "Modern Architecture," published by the N. Y. Museum of Art in 1932, contains pictures of the International Exhibition of Modern Architecture. "New World Architecture" is a fascinating book with 389 illustrations showing how architecture has changed with civilization. There is a book called "Essentials of Design" which discusses ancient and modern conceptions of industrial arts, principles of designing, home decorations, etc. "New Dimensions, the Decorative Arts of Today in Words and Pictures" contains illustrations of decorative fabrics, modern furniture and modern art in business. "Landscape Gardening" is a treatise on the general principles governing outdoor art with suggestions for their application in the common problems of gardening, such as planning school grounds, suburban lots, and parks.

There is so much material to read concerning the use of geometry in industry that I will mention only a few of the books I have on my list in order to illustrate the general types of material. "Building a Model Railroad" is written for boys and explains tracks, engines, and power supply. "The Romance of Modern Engineering" is a fascinating book, containing descriptions of pictures of the Nile Dam, the Panama Canal, Brooklyn Bridge, etc. "With the Men who do Things" is meant to interest boys in engineering with romantic stories such as "Five Hundred Feet Above Broad-

way," "One Hundred Feet Below Broadway," and "Spinning a Web Across the River."

But we wouldn't want boys to be engineers if they haven't the talents necessary. The prerequisites necessary for successfully carrying on an engineering course of study and the actual work and duties of an engineer in practice are not generally understood outside of the profession itself. As a consequence, many young men select engineering as a career who later find a change necessary or advisable. There is much material which will describe what kinds of abilities are needed for various kinds of engineering. This furnishes the teacher a chance to do some work in vocational guidance. The United States Department of the Interior has Guidance Leaflets on such subjects as Architecture and Mechanical, Chemical, Civil and Electrical Engineering. Carnegie Institute of Technology published a bulletin called "Engineering as a Profession." By far the most valuable material I have found on the subject is "Vocational Guidance in Engineering Lines" edited by the American Association of Engineers.

Students who like geography may like the study of circles in connection with longitude and time, map projections, and circumnavigation and time as given in "Mathematical Geography." There is a book called "Occupations" which explains the uses of geometry in agriculture, manufacturing and transportation. "Paths to Success" is a series of sixteen essays on secondary school subjects, the one in mathematics discussing geometry as the earth-measurer. Students who like history but think mathematics useless might gain an appreciation by reading the article published in *Scripta Mathematica* and written by D. E. Smith on "Thomas Jefferson and Mathematics."

Of course, before any of this unit can be organized the teacher must have had his own bibliography. I haven't even mentioned that because each person will choose that for himself according to his own needs. However, I believe you will find that you will gain much from the readings you place on the student's bibliography.

You may be interested in the sources of the illustrative material which I have found. Some of the illustrations of geometry in nature were bought from the American Museum of Natural History in New York City and some showing geometry in art were purchased from the Metropolitan Museum of Art. However, by far the most of the pictures were cut from newspapers, magazines, and cata-

logues. Smithsonian Institute in Washington, D.C., will furnish material on the Maya Indians showing primitive ornament. The National Geographic Society sent the chart of Byrd's flight free of charge. Many pictures may be secured by writing for catalogues. I wrote to a certain automobile company and asked for a catalogue showing their latest models in cars. My hope was that I might find pictures of some of the machinery inside the car. By return mail the company not only sent a catalogue but wrote a letter saying they would be glad to send a salesman to see me at any time. You will be surprised at the prompt reply you will often get.

The Mack Printing Company of Easton, Pennsylvania, sent cuts of some of the illustrations used in their book "Vocational Guidance in Engineering Lines" and the printing department of our own high school printed them. So without any cost I have a picture of the Hoover Dam across the Colorado River as an example of dam-engineering; a diagram of the inter-district traffic circuit in Pittsburgh, Pennsylvania, as an example of city-planning engineering; one of the Columbia Highway at Shepherd's Dell, Oregon, illustrating highway engineering; one of the Horseshoe Curve on the Pennsylvania Railroad as an example of railway engineering; the Empire State Building illustrating steel-construction engineering; a picture showing congested traffic lines at 33rd Street in New York City as an example of subway engineering; and a diagram showing overseas telephone connections of the American Telephone and Telegraph Company as an illustration of telephone engineering.

While I have been describing all this you have probably been wondering just what all this has to do with geometry. To be sure, some of it is quite far removed from the geometry of the past. We may as well realize that many of our students will never appreciate geometry as an example of postulational method of thinking. But if by some picture or by some reading material we could somehow reach every student enough to convince him that the subject he is studying is not a dead subject, that the world could not in the past nor cannot in the future get along without it; if we could get each student interested in something, whether for a vocation or an avocation, which he might follow later either in or out of school, then our time has been worth while. It is true that all this takes time in the class room but even if the student gets less information he may get more inspiration for the future. I like what Professor

Hedrick said: "What does it profit a man if he learn every fact and acquire every skill of mathematics if he loses the soul of the subject?"

Our textbooks have been patterned too closely after the model of Euclid whose text was not intended for the use of boys and girls but for mature men. They are beginning to change; our newest texts contain many more illustrations and much more practical material than formerly. But after all, the greatest responsibility lies with the individual teacher. The teacher must inspire interest in the subject or it is dead and to do this the teacher must be interested himself. At a recent meeting of mathematics teachers there was a panel discussion in which one point taken up was that the teacher should "generate" enthusiasm for his field. I thought it would have been better if they had said "feel" enthusiasm for his field. We can't generate it unless we feel it.

Glenn Frank, in his Prayer for Teachers, says "Help us to be more interested in stimulating builders of modern cathedrals than in retailing to students the glories of ancient temples." I am not sure just how much he meant by that statement but I am sure that if we as teachers could have the philosophy and the vision to feel that we are stimulating builders of modern cathedrals in the broadest sense of the word, then mathematics would take its place beside social science in contributing to the general aim of our educational scheme—preparation for life.

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Pierre-Simon Laplace

Born at Beaumont en Auge, March 23, 1749
Died in Paris, March 5, 1827

THE WORK OF Laplace is of such an advanced character that it is prudent in this sketch to concentrate on the details of his career and his character and to give only brief mention of his contributions to mathematics.

His life was closely tied to French politics. He was born in Normandy in the reign of Louis XV. His family was poor and it was only through the interest of wealthy neighbors that Laplace was enabled to attend the school in Beaumont. At the age of eighteen, he went to Paris with letters of introduction to d'Alembert who was then at the height of his career. D'Alembert ignored the letters but his interest was aroused by a paper on mechanics which Laplace sent him at a later date. D'Alembert then had Laplace appointed as professor of mathematics at the Ecole militaire in Paris. Under normal circumstances, Laplace might have anticipated continuing in this position and later in the post of examiner of the military corps, to the end of his days. He took advantage of the security his position afforded and made many contributions to astronomy in the period from 1771 to 1787. Laplace was forty in the eventful year 1789. The years of his life, almost forty, covered the period of the various revolutionary governments in France, the Napoleonic era and better than a decade after the restoration of the Bourbons. Throughout this time, Laplace kept in the favor of each ruling party in turn.

It will be noticed, however, that Laplace was primarily a scientist and in consideration of the period in which he lived, one should be guarded in condemning his frequent shifts of allegiance. When a reform of the calendar was in progress, an astute politician would have suggested the beginning of the French Revolution as the proper time for the starting of a new era. Laplace urged that the years be numbered from the vernal equinox of the year 1250 A.D. when he computed that the major axis of the earth's orbit was perpendicular to the equinoxial line. Needless to say, the starting point chosen for the era coincided with the creation of the Republic.

Laplace was a member of the commission on the metric system. He had a part in the development of the Ecole normale and the Ecole polytechnique. The first of his three great works was his *Exposition du système du monde* published in 1796 and dedicated to the Council of 500. This volume contained the nebular hypothesis as well as a summary of the history of astronomy and it earned for its author a coveted place in the French Academy which then had but forty members.

When Napoleon came into power, Laplace begged for a government appointment and was made minister of the interior. He was speedily dismissed, Napoleon making this comment.—"A geometer of the first rank, Laplace was not slow in showing himself as a more than mediocre administrator. From the first of his work, we recognized that we had made a mistake. Laplace could comprehend no question in its true light: he searched out every subtlety, . . . and he carried the spirit of the 'infiniment petits' even into administration." In compensation for removing him from his post, Napoleon made Laplace a member of the senate and gave him a title.

The second of the great works of Laplace was his *Mécanique celeste* which appeared in five volumes, two in 1799 and the others in 1802, 1805, and 1825. Ball says of this that "the *Mécanique celeste* is not only the translation of the *Principia* into the language of the differential calculus, but it completes parts of which Newton had been unable to fill in the details."[1] Biot who assisted Laplace in preparing the manuscript for publication, says that when Laplace came to an hiatus in the chain of reasoning and yet was satisfied that the conclusion was correct, he would cover the gap with the statement, "It is easy to see." And the American mathematician Nathaniel Bowditch who translated the work into English admitted that at each of these remarks, he knew hours of work would be needed to reconstruct the reasoning.

It was characteristic of Laplace that the introduction of the third volume of the *Mécanique celeste* contained a declaration of the author's devotion to Napoleon as the peacemaker of Europe. It was also characteristic of him, that this was suppressed in editions printed after 1814.

When Laplace presented a copy of the *Mécanique celeste* to Napoleon, it is said that the emperor took Laplace to task for writing so great a volume on the system of the universe without

[1] Ball, W. W. R. *Short Account of the History of Mathematics*, 1915 ed., p. 417.

mention of the Creator. According to the story, Laplace replied, "I had no need of that hypothesis." Cajori reads into the incident a side of the story that does not appear at first glance. He says,

> I. Newton was not able to explain by his law of gravitation all questions arising in the mechanics of the heavens. Thus, being unable to show that the solar system was stable, and suspecting in fact that it was unstable, Newton expressed the opinion that the special intervention, from time to time of a powerful hand was necessary to preserve order. Now Laplace thought that he had proved by the law of gravitation that the solar system is stable, and in that sense may be said to have felt no necessity for reference to the Almighty.[2]

The third great work of Laplace was his *Théorie analytique des probabilities* (1812). This contained the method of least squares.

In 1814, Laplace voted against Napoleon and Loria notes that he did not present himself to Napoleon during the Hundred Days. It was probably prudent not to. Recognizing the apparent loyalty of Laplace to the Bourbons, Louis XVIII made him a peer of France and gave him the title of marquis.

Ball summarizes the personality of Laplace as follows:

> That Laplace was vain and selfish is not denied by his warmest admirers; his conduct to the benefactors of his youth and his political friends was ungrateful and contemptible; while his appropriation of the results of those who were comparatively unknown seems to be well established and is absolutely indefensible. . . . On the other side, it may be said that on some questions he shewed independence of character, and he never concealed his views on religion, philosophy, or science, however distasteful they might be to the authorities in power; it should also be added that towards the close of his life, and especially to the work of his pupils, Laplace was both generous and appreciative, and in one case suppressed a paper of his own in order that a pupil might have the sole credit of the investigation.[3]

VERA SANFORD

[2] Cajori, Florian. *History of Mathematics*, 1926, p. 262.
[3] Ball, *loc. cit.*, p. 421.

Abstracts of Recent Articles on Mathematical Topics in Other Periodicals

By NATHAN LAZAR
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Algebra

1. Linn, Marguerite. *An experimental comparison of two methods of teaching elementary algebra.* School Science and Mathematics. 34: 983–5. December 1934.

Résumé of a master's thesis presented to the Faculty of the School of Education, University of Southern California. The two methods chosen for comparative study were (1) "the one known as the traditional recitation method, and (2) a modified form of the unit plan. The recitation method, as used, consisted, in brief, of uniform daily assignments, daily consideration of the previous assignment, discussion of difficulties, and class drill, with occasional tests. The modified unit method consisted of a group meeting at the beginning of each part of a unit; several days devoted largely to laboratory work, or supervised study; and a group meeting at the end of each unit for the purpose of discussion and testing. Graded assignments were used, each student choosing the group in which he wished to work."

The procedure consisted in equating as nearly as possible, two groups of thirty-two students each, all of whom were without previous experience in the field of elementary algebra. One group was taught during the remaining thirty weeks of the school year by the recitation method while the other was taught by the unit plan. The groups were equated on the basis of the scores obtained on three standard tests administered during the first six weeks: (1) the Terman Group Test of Mental Ability, (2) the New Stanford Arithmetic Test, and (3) the New Stanford Reading Test.

Achievement was measured by means of the Douglass Standard Survey Tests 1 and 11, Form A, and by twelve supplementary tests, which served as a check on the results of the standard tests.

The conclusions were:

1. There was little difference between the results obtained in the tests on the mastery of subject matter.

2. The students themselves, however, preferred the unit plan. The ratio of preference increased in direct proportion to the grades—that is, the higher the grade the greater the ratio of preference for the unit method.

3. There are other subjective factors, which might be called character-builders that the unit plan tends to foster in the student; such are: "independence of thought and action, initiative, intellectual honesty, and self-dependence. That these are valuable contributions to the education of the student cannot be doubted."

Since the article summarized is itself a résumé of a larger paper it would be unfair to cavil at certain omissions. But one reader, at least, would like to know whether Miss Linn asked the pupils the reasons for their preference for the unit method. For, it is conceivable that such

opinions, if frankly given, might cast doubts upon the claim that the unit plan inculcates the desirable traits enumerated above.

2. Miller, Franklin, Jr. *A practical three-dimensional graph.* School Science and Mathematics. 34: 919–22. December 1934.

"The purpose of this paper is to describe a fairly simple and convenient method for the construction of three-dimensional graphs. The construction makes use of calculated or observed contour-lines and in the final result the contour-lines are very much in evidence."

The procedure is then described in great detail, with the help of two diagrams and one photograph.

This kind of graph can be used to great advantage in the experimental sciences and in mathematical economics where it is often desired to graph a quantity which depends upon two independent variables.

3. Read, Cecil B. *An approximate solution of a system of quadratic equations.* School Science and Mathematics. 34: 942–44. December 1934.

The writer describes a method of solving a system of quadratic equations, such as the following, to any desired degree of accuracy, without the use of Horner's or Newton's method:

$$x^2+y=5$$
$$x+y^2=6$$

It is of course evident that the methods usually taught for the solution of quadratic equations are not applicable to the above system.

To illustrate the new method, let us take the example given above. "Even a crude graphical solution gives the following approximate values: $(-1\frac{1}{2}, 3)$; $(-3, -3)$; $(2\frac{1}{2}, -2)$; and $(2, 2)$. Let us concentrate on the approximate solution $(2, 2)$. Suppose the actual solution is $(2+a, 2+b)$ where a and b are corrections. Since these are small quantities, terms involving them which are of a second or higher degree may be neglected. If we substitute these values in the original equations and neglect second degree terms, we have the linear system

$$4a+b=-1$$
$$a+4b=0$$

The solution of this system, computing results to two decimal places, is $a=-.27$, $b=.07$. Accordingly, as a second approximation, we have the solution, $x=1.73$, $y=2.07$.

"If greater accuracy is desired, the process is repeated, using these values for x and y and assuming the actual value to be $(1.73+a_1, 2.07+b_1)$, where a_1 and b_1 are supplemental corrections.

"Obviously the method can be applied to any desired real root. In any physical application a good guess will usually supply the first approximation, eliminating the need for a graph."

4. *The trianalyst.* Scientific American. 151: 329–30. December 1934.

This article contains a photograph and a description of a mathematical device, the invention of a lawyer, Barret R. Wilmington of Troy, New York. It is claimed that it can solve numerical algebraic equations—quadratics, cubics, biquadratics and certain forms of simultaneous equations.

The device depends, for its operation, upon the following geometric theorem: "The area of a square inscribed in a right triangle is equal to the product of the legs of the right triangle each diminished by the edge of the square."

The above theorem can easily be shown to be true. But, unfortunately, the other relevant information given in the article is far too meager, for this reviewer at least, to enable him to

formulate a complete explanation of the theoretical foundations and the practical efficiency of the instruments.

5. Woodruff, J. Shaylor. *Euclidean construction for imaginary roots of the quadratic equation.* School Science and Mathematics. 34: 950–57. December 1934.

"The method most often employed in our modern algebra for the graphical solution of the quadratic equation involves the construction of the parabola. As the parabola enables us to find the roots, when real, of any quadratic equation by suitable movement of the axes, there seemed some warrant for the conclusion that an intimately related curve would perform a similar function for imaginary roots, and make the graphical representation complete." Such a method has indeed been devised and the name given to the auxiliary graph, after Robert L. Stevenson's famous poem, was "shadow curve."

The writer of the article believes, however, that "although the parabola and shadow parabola readily give the solutions, the circle has the advantages in that it is more easily and accurately drawn and we can limit ourselves to the ruler and compasses. Thus we have the satisfaction of developing a construction within the restrictions of Plato's dictum—use only the rule and compasses." Accepting this dictum and introducing a shadow circle, he presents a method by which imaginary roots of the quadratic equation can be found graphically as easily and as accurately as real roots. The construction for imaginary roots will follow closely in the steps of that for real roots, justified too with an almost identical proof.

A clear and detailed description of the method is given, illustrated by five diagrams. A full bibliography is also appended.

The procedure, as well as the proofs, are well within the grasp of the brighter members of a class in Intermediate or Advanced Algebra, to whom it may be assigned as extra-curricular activity. It may also be used as an interesting diversion on the program of a mathematics club.

Arithmetic

1. Buswell, G. T. *Selected references on instruction of arithmetic.* The Elementary School Journal. 35: 212–14. November 1934.

A bibliography, with short descriptive annotations, of the literature on that topic, published during the period from July 1933 to June 1934 inclusive. "The writer has used as his basis of selection chiefly the presentation of new material, of the serious discussion by competent authorities of problems for which no new data were available."

2. Dickey, John W. *The value of estimating answers to arithmetic problems and examples.* The Elementary School Journal. 35: 24–31. September 1934.

The report of a carefully conducted experiment to determine the validity of the commonly accepted opinion of experts in pedagogy that the advance estimating of answers improves the ability to solve problems. The conclusion is "that the problem-solving ability gained by the sixth grade pupils as a result of practice in estimating answers to arithmetic problems is neither more nor less than that gained through practice in solving problems in the traditional manner." The discussion of the problem, the technique devised for solving it, and the analysis of the limitation of the conclusions are all models of strict scientific procedure. Experimentation of such a nature, although it yields but tentative results, would do much to

increase the regard in which educational experimentation is usually held.

3. Hughes, Harold F. *Character training from the arithmetic lesson.* Educational Method. 14: 93–96. November 1934.

A detailed analysis of the many ways in which "number drill may be taken out of the drudgery category and made a force in character training." By the term *character* the writer does not mean anything nebulous or enigmatic; he has reference to specific traits and attitudes that are amenable to the laws of habit formation, such as

a. assuming personal responsibility for individual progress,

b. giving help to those who know less, and

c. receiving, in return, assistance from those who know more.

Miscellaneous

1. Barnett, I. A. *A proposal for the improvement of the teaching of mathematics.* National Mathematics Magazine. 9: 74–81. December 1934.

The chairman's address at the annual meeting of the Ohio Section of the Mathematical Association of America held in Columbus, Ohio, April 5, 1934. The writer has faith in the possibility of improving the training of teachers of mathematics and proceeds to give a detailed description of a course that will bring about the desired results.

2. Braverman, Benjamin. *The mathematics exhibit at the Century of Progress Exposition.* High Points. December 1934. Pp. 30–31.

A brief and enthusiastic description of the mathematical devices and machines exhibited at the recent exposition at Chicago, and of the evident interest and delight shown by the attending crowds.

"A teacher of the subject leaves this exhibit with newer and clearer ideas of how models and other illustrative material can be used to vitalize and make more interesting the teaching of mathematics. Also with the wish that it were possible to obtain slides of the entire exhibit in order that our pupils could thrill to the beauty and glory that belong to mathematics in the same way as the crowds at the Fair were thrilled."

3. Harris, Louise M. *Exhibit your mathematics.* High Points. December 1934. Pp. 22–25.

A description of a project carried out by the department of mathematics of the Manual Training High School, Brooklyn, New York.

Towards the end of the term a room was set aside in which the following were placed on exhibition: (a) mathematical instruments (a transit, a sextant, and a slide rule); (b) superior homework papers, test papers and notebooks; (c) charts and graphs; and (d) models.

"The list following is but a partial one of the topics which we used, but it will give the reader some idea of a point from which to start.

I. Elementary algebra.
   - A. Graphs
     1. Circle, line, bar—made by pupils using data of interest to them.
     2. Illustrations from newspapers and magazines.
     3. Solutions of sets of linear equations.
     4. Solution of problems.
   - B. History of algebra. Short stories and pictures of famous men influential in the development of algebra.
   - C. Simple algebraic fallacies.
   - D. Practical problems in numerical trigonometry.

II. Plane geometry.

A. Models, diagrams and instruments made by students to illustrate various theorems in the course.
B. Geometric designs. Geometry in art and in nature.
C. The nine-point circle. Short explanation and description.
D. Geometry of the pyramids of Egypt.
E. The three problems of antiquity.
F. Four color map problem.
G. Methods of proving the Pythagorean proposition.
H. Meaning of π; its value to a given number of decimal places.
I. Ceva's theorem; its uses and other facts of importance.

III. Intermediate algebra.

A. Graphs—various conics; illustration of Kepler's laws, and relations of orbits, comets and planets to conic sections.
B. Pascal's triangle; its use in the binomial theorem.
C. Use of geometric progression for business-compound interest, annuities, etc.
D. Cube root.
E. Puzzles.
F. Fallacies.

IV. Plane trigonometry.

A. The slide rule
B. The transit
} Actual models and directions for their use.

C. Trigonometric instruments used at sea.
D. Graphs of trigonometric and inverse trigonometric functions.
E. Graphs of hyperbolic functions.

V. Advanced algebra.

A. Compound interest graphs.
B. The Cardan-Tartaglia controversy.
C. The application of probability to hands of bridge.
D. The relationship among the constants i, e, and π.
E. A set of paper models illustrating a problem in maximum and minimum.

VI. Solid geometry.

A. Models of the five regular polyhedra.
B. Models of all the conic sections as actual sections of a cone.
C. Pen and ink drawings of the figures for text-book propositions.
D. Model of a regular pyramid inscribed in a cone, and of a cone inscribed in a regular pyramid constructed so as to show the relationship between slant height, elements and lateral edge."

Lists indicating the sources of materials and books on reference are appended.

4. Orleans, Joseph B. *Newer objectives in teaching mathematics in the secondary schools.* Secondary Education. 3:153–55. September 1934.

A report, by the chairman of the Standing Committee on Mathematics in the New York City High Schools, of the proposed revision of the course of study. The committee believes in the importance of mathematics in the curriculum of the secondary school because it believes that mathematics is the "frame of the universe." In order to determine the content of the course of study it asked itself the following questions: "What desirable changes should be brought about in the pupil through a study of high school mathematics? In what respects should a boy or girl at the end of a year of algebra or of geometry or, better still, of comprehensive mathematics, be different for the better from what he was at the beginning of the year because of the study of the subject?

"The answers to these questions were to determine the content of the course of study. Any topic which cannot be justified on the basis of them must be omitted. The committee then drew up the following list of ideals to be realized in the growth of boys and girls as the result of the teaching of mathematics in the secondary school.

1. The will to perfection.
2. Effective habits of work.
3. The habit and power of analysis.
4. Open-mindedness.
5. Courage for truth.
6. The habit of reflection.
7. Freedom in thought as the outcome of a long course in mental discipline.

8. Reverence.

"The committee recognizes that each of these ideals can be realized also somewhat through other subjects; but it feels that the realization can best be brought about through the learning of mathematics just as other ideals can best be realized through the social sciences or the manual arts.

"The committee then listed the following immediate objectives in *learning* mathematics:

1. Appreciation of the value of symbolic representation.
2. Extension of the number concept.
3. Appreciation of functionality.
4. Analysis.
5. Graphic representation.
6. Spatial imagination.
7. Effective thinking.
   a. Organization of thinking.
   b. Precision of expression.
   c. Intelligent guessing.
8. Confidence in reasoned conclusions.
9. Esthetic and spiritual values.
   a. Appreciation of mathematical law in science and of mathematical forms in nature and in art.
   b. Reverence as an outcome of contemplation.

"With these lists as a guide the committee proceeded to prepare a comprehensive course in the mathematics of the ninth and tenth years. . . . This comprehensive course is now in tentative form and is being tried out in several of the high schools in New York City."

The proposed revision of the mathematical syllabus in the largest city of the country bids fair to become one of the major events in the history of mathematical education in the United States. Moreover, the beneficial psychological effects should not be overlooked. The actuality of bringing about a fundamental change in an educational system as complicated as that of New York City, should serve as an inspiration to weary teachers and pessimistic educators the country over!

5. Schlauch, William S. *Meeting community needs through the teaching of mathematics in high schools.* Secondary Education. 3:144. September 1934.

What teachers in the high schools of the United States need to do, to measure up to the task before them may be summarized as follows:

1. "Minimize or drop such algebraic techniques as have no application in the fields of business, finance and the professions.
2. "Add to the curriculum in mathematics subject matter which does so function as probability, insurance statistics, annuities, and their applications.
3. "Study the fields in which mathematics is scoring its greatest triumphs, and which are most useful

to the citizen: economics, statistics, accounting, mechanics, physics, finance, navigation and astronomy.

4. "Give illustrations and practical problems taken from these fields to illustrate the use of algebraic processes and techniques, instead of using the old traditional material and problems."

6. Wren, F. L. *The scholarly teacher of mathematics.* The Kadelpian Review. 14:20–28. November 1934.

The writer regrets the influx of teachers who are "certified" but "not particularly interested" in their chosen subject and who can do no more than merely guide and direct in the mechanics of mathematics. It is to that condition that he attributes the harsh and severe criticism to which mathematics has, of late, been subjected. He then proceeds to outline a course of study for a prospective teacher, pointing out the high spots in each of the branches of mathematics.

He believes that "mathematics, luxuriant in its historical significance as a leader in scientific progress, technological efficiency and philosophic exactness, is replete with present realities, and pulsates with future potentialities. The mathematical preparation of the teacher of secondary mathematics should provide that historical, technical and liberal information that would enable him to so effectively organize and present the subject matter, that adolescent boys and girls should be confronted with many opportunities to do constructive thinking, to realize the intrinsic worth of mathematics as an interpreter of his environment and as a contributor to their more efficient functioning as members of a civilized social order, and to come into contact with logical processes that should better enable him to discriminatingly integrate those principles and truths which tend to make living more significant and from which he might evolve a wholesome philosophy of life."

All of these plans for the improvement of the breed of the *coming* generation of teachers of mathematics are admirable and desirable. One can not have too much of intelligent planning. But often, too often, one can detect a suppressed feeling that the *present* generation of actively engaged teachers is beyond redemption and that it is profitless to bother about them. On the practical side this attitude is unwise. For there may not be need for a second generation of teachers of mathematics if the teaching of the subject is as bad as so many observers claim. On the theoretical side this attitude goes counter to the conclusions drawn by Prof. Thorndike in his volume on "Adult Education" that the mature adult is just as competent to learn new tricks as the fledglings.

It might be an interesting experiment to call a moratorium, for a decade or two, on such vague exhortations as "teach the spirit of mathematics," "inculcate the beauty of deductive reasoning," and "show the pervasiveness of the notion of functionality," and to demand in their places concrete, specific, detailed directions for actual class room procedure. Current philosophic theory as well as intelligent pedagogic practice are beginning to suspect the meaningfullness of ideals which can not be "operationally" interpreted.

Final Program for the Sixteenth Annual Meeting of the National Council of Teachers of Mathematics

Chelsea Hotel, Atlantic City, N. J.
February 22 and 23, 1934

11:00 A.M. Business Meeting of the Board of Directors.

12:30 P.M. Luncheon Meeting of the Board of Directors.

2:00 P.M. Business Meeting of the Council. (Open to all members)

1. Reports of standing committees with a discussion of educational policies.
2. Election of officers.

6:00 P.M. Dinner for directors and official delegates of branches and affiliated organizations.

8:00 P.M. First Program Meeting.

1. Address of Welcome
   Dr. Frank J. McMackin, Jersey City, N. J.
2. Response.
   Wm. Betz, Rochester, N. Y.
3. *The Psychology of the Transfer of Training.*
   Dr. P. T. Orata, Ohio State University.
4. *Methods of Teaching for the Maximum Amount of Transfer.*
   Harold P. Fawcett, University High School, Columbus, Ohio.

Saturday, February 23

9:30 A.M. Second Program Meeting, Vice President Allen R. Congdon presiding.

General Theme

"*Crucial Problems in the Field of Arithmetic Teaching.*" C. L. Thiele, Director of Exact Sciences Detroit Public Schools—Chairman

1. "*Bases for Selecting Curricular Content Related to the Social Aspects of Arithmetic*" (Fifteen minutes)
   Dr. Leo. J. Brueckner, Professor of Education, University of Minnesota.

2. "*What Research Reveals on Proper Drill Content of Elementary Arithmetic*" (Fifteen minutes)
   Dr. Guy M. Wilson, Professor of Education, Boston University.
3. "*Non-Computational Arithmetic*" (Fifteen minutes)
   Dr. G. T. Buswell, Professor of Educational Psychology, Department of Education, University of Chicago.
4. "*Growth Stages and the Curriculum*" (Fifteen minutes)
   Dr. C. Washburne, Superintendent of Schools, Winnetka, Ill.
5. "*Arithmetic in the Primary Grades*" (Fifteen minutes)
   Dr. Saul Badanes, Principal, Public School 173, The City of New York.
6. "*Some Current Issues in the Teaching of Arithmetic*" (Fifteen minutes)
   C. B. Upton, Professor of Mathematics, Teachers College, Columbia University.
7. "*A Proposed National Committee on Arithmetic*" (Ten minutes)
   Dr. R. L. Morton, Professor of Mathematics, College of Education, Ohio State University.

Round Table Discussion

12:00 P.M. Joint luncheon meeting of the new and old boards of directors.

2:00 P.M. Third Program Meeting.
General Topic—The Subject We Teach.
Arranged by first vice president R. D. Beatley.

1. *The Geometry of Inversion.*
   Professor Roger Johnson, Brooklyn College Brooklyn, N. Y.
2. *Graphical Methods in Mathematics.*
   Professor George W. Mullins, Barnard College, Columbia University, New York, N. Y.

6:30 P.M. Annual Banquet. The honored guest and speaker at the banquet will be Professor Cassius J. Keyser of Columbia University whose topic will be *Mathematics and the Dance of Life.*

Rates at the Hotel Chelsea

The rates for rooms at the Hotel Chelsea are as follows: Single rooms, without bath—$2.50, $3.00, and $400; with bath—$3.00, $4.00, and $5.00. Double rooms, without bath—$4.00, $5.00, and $7.00; with bath—$5.00, $6.00, $7.00, and $8.00. Write at once for reservations.

NEWS NOTES

PROFESSOR EARLE R. HEDRICK of the University of California at Los Angeles will give two courses in Mathematics at Teachers College, Columbia University this summer. The first course will be on "Teaching Mathematics in Junior Colleges and in Lower Divisions of Colleges and Universities" and the second on "Professionalized Subject-Matter in Algebra and Geometry." A more complete description of the nature of the above courses will appear in a later issue of *The Mathematics Teacher*.

DR. JOHN L. TILDSLEY, district superintendent in charge of New York City High Schools was the guest speaker at the last meeting of the Association of Mathematics Teachers of New Jersey at Newark on January 12th, 1935. His topic was "Some Observations and Deductions of the Present and Future Position of Mathematics in the High School."

To mark the seventy-fifth annual occurrence of the birthday of David Eugene Smith, the staff of Columbia University Library held a dinner on January 21, 1935 at the Faculty Club. Old friends, colleagues, and former students of Professor Smith were present and several short, but appreciate speeches on the life and work of Professor Smith were given. Professor Reeve of the Mathematics department of Teachers Colege, Columbia University spoke as follows:

"It is particularly fittting that this anniversary dinner to Professor David Eugene Smith has been sponsored by Mr. Howson and his Library group, because books have been the constant companions of our honored guest ever since he was old enough to read. As a small boy, when left by his father to hoe the weeds from the garden, he found reading a book on top of the garden fence more to his liking. His love of books has enabled him to acquire a wide familiarity with most of the achievements of the human spirit—the intricate points of fishing and golf being rare exceptions.

"He has been not only a lover of books—a consumer, so to speak,—but he has been also a producer, giving joy to hundreds of children through such books as 'Number Stories of Long Ago,' and inspiration to scores of teachers through his many essays like 'Religio Mathematica.'

"But we honor Professor Smith tonight not so much because of what he has done, as for what he is; what he has meant to those of us who have come in contact with him. The most important things his students have ever obtained from his courses were 'caught, not taught.' In fact, there is something remarkable about the enthusiasm and devotion of students who have had the good fortune to know him as teacher and friend.

"It has been my privilege to know Dr. Smith in a rather intimate way for a great many years. As a wielder of the blue pencil he has no equal, as counselor in time of uncertainty and adversity he has always been sympathetic and helpful, and as a friend he is true blue.

"We know that he has done many things of real worth that we cannot yet appreciate. Perhaps he himself does not

realize what he has actually accomplished. We can only say with Browning:

"There shall never be one lost good!
What was shall live as before;
The evil is null, is naught, is silence
implying sound;
What was good, shall be good, with for
evil, so much good more;
On earth the broken arcs, in the heaven,
a perfect round."

THE Men's Mathematics Club of Chicago and the Metropolitan Area, opened their twenty-first season on Friday, October 19, 1934. The officers of the club for the coming year are Mr. J. J. Urbancek, president; Dr. W. C. Krathwohl, secretary-treasurer; Mr. J. R. McDonald, recording-secretary. At the first meeting Dr. C. A. Stone gave a paper on "The Status of Solid Geometry in Secondary Mathematics," in which he showed how plane and solid geometry could be combined in such a way that each clarified difficulties in the other. Dr. Georges gave a paper on "The Operator J and Errors in the Learning of Mathematical Concepts and Processes." He explained a method which could be used to analyze many common errors of beginners in algebra.

Three papers on methods of teaching, were given at the November meeting. Mr. J. A. Nyberg gave a paper on "The Teaching of Algebra" in which he exhibited a method which he found highly successful in teaching verbal problems. Mr. Charles Leckrone's paper on "The Teaching of Geometry" showed how in beginning geometry, he aroused interest by appealing to the pupil's imagination. Mr. C. M. Austin in his paper on "The Teaching of Trigonometry" explained how he was able to unify the course and stimulate independent investigation.

At the December meeting, Major Chester L. Fordney spoke on "My Trip to the Stratosphere with Lieutenant Commander Settle." He told how mathematics was used in preparing for the famous November flight, how it was used during the flight, and how it was used afterward in interpreting the results of some of the observations.

MCGRAW-HILL Book Company have recently published a neat little booklet to commemorate the 25th anniversary of the founding of their business. Articles on "The Power in Books," photographs of members of the firm and their editors and authors, and other interesting features appear.

THE ANNUAL Spring meeting of the Mathematics Section of the California Teachers Association, Bay Section, was held Friday, May 11, 1934, at 8:15 p.m., in Room 210 of Le Conte Auditorium on the University of California Campus, Berkeley.

D. N. Lehmer, Professor of Mathematics, University of California, Berkeley, presented an "Exhibition of Three-Space Curves by the Pollock Models" and gave an interesting discussion of Mathematics at the World's Fair in Chicago.

A short business meeting was held for the election of officers for the year 1934–35. They are:

Chairman—Mr. Walter Fredrickson, High School of Commerce, San Francisco

Vice Chairman—Miss Alma Adams, Roosevelt High School, Oakland

Secretary-Treasurer—Miss Una McBean, Horace Mann Junior High School, San Francisco

(Mrs.) NINA H. SLEWING, Secretary

THE PROGRAM of the Detroit Mathematics Club for the year 1934–35 is as follows:

November 15, 1934

"The Mathematics of Alice in Wonderland," MRS. DOROTHY VAN DEUSEN, Head of Mathematics Department, Battle Creek High School.

Tea 3:30 P. M. Address 4:30 P. M.

February 7, 1935

"Teaching Geometry by the Discovery Method," MR. ENOS PORTER, Chadsey High School.

"Mathematics and the Slow Pupil," MISS HILDA BECK, McMichael Intermediate School.

March 21, 1935

"Meaningful Learning of Plane Geometry," PROFESSOR H. C. CHRISTOFFERSON, School of Education, Miami University, Oxford, Ohio.

Tea 3:30 P. M. Address 4:30 P. M.

May 2, 1935

"The Challenge to Our Taxation Policies," PROFESSOR S. M. LEVIN, Head of Department of Economics, Accounting and Sociology, Wayne University.

The officers of the Club are:

President: Byron E. Smith, Post Int. School.

Vice-Pres.: Verna B. Philbrick, Hutchins Int. School.

Secretary: Evelyn Dougherty, Cooley High School.

Treasurer: Wade Hulbert, Northwestern High School.

THE PITTSBURGH MEETING OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

December 28–29, 1934

Edwin W. Schreiber, Secretary

THE MEETINGS of the National Council of Teachers of Mathematics were held in conjunction with those of the American Mathematical Society, the Mathematical Association of America, and Section A of the American Association for the Advancement of Science. This is the first time that the National Council of Teachers of Mathematics has met with the American Association for the Advancement of Science. All sessions were held at the Carnegie Institute of Technology except the joint banquet which was held in the Georgian Room of the Hotel Webster Hall on Saturday evening, December 29, 1934. One hundred thirty-six were registered.

The opening session was called to order by President J. O. Hassler of the University of Oklahoma and began at 8 o'clock on Friday evening, December 28, by an address of welcome given by Professor Edwin G. Olds of the Carnegie Institute of Technology. The response was made by Professor W. D. Reeve of Columbia University. Professor Reeve spoke at some length on certain topics of curricular revision in which the audience expressed considerable interest. An interesting feature of the evening was a Symposium on Methods of Making Mathematics Interesting, by a group of teachers from the Pittsburgh High Schools and directed by Dr. Elizabeth B. Cowley. Those taking part were Isabel Epley, H. W. Graffius, John S. Kelso, Bertha J. Kirkpatrick, Harry E. Long, Truman C. Cheeseman, Nan W. Reniers, and Edna E. Schlegel.

The Saturday morning session was devoted to the general topic, "Mathematical Concepts of Value to High School Teachers." Professor H. W. Brinkman of Swarthmore College presented a very interesting paper on "Certain Concepts in Trigonometry." Professor C. C. MacDuffee of Ohio State University presented very logically his paper on the "Different Kinds of Equality." A special feature of the morning session was a display of English text-

books and examination papers collected by Professor W. D. Cairns, secretary of the Mathematical Association of America. Professor Cairns collected this material while visiting English Schools on a recent trip abroad. The exhibit which included various charts of typical problems used for examination purposes in England together with the standard textbooks used in Britain were available all day Saturday.

The Saturday afternoon session was a joint meeting with the Mathematical Association of America on the topic "The Need for a Re-Orientation of Mathematics in the Secondary Schools." The session was called to order by Professor Arnold Dresden, President of the Mathematical Association of America, who welcomed the National Council of Teachers of Mathematics to this first joint session of the two mathematical organizations. President Hassler introduced the speakers of the afternoon. The first paper entitled, "From the Viewpoint of Modern Educational Theory," was presented by Professor P. W. Hutson of the University of Pittsburgh. Professor W. L. Hart of the University of Minnesota presented the second paper entitled, "From the Viewpoint of the University Professor of Mathematics." Dr. M. L. Hartung of the University High School, Madison, Wisconsin, presented the third paper on "From the Viewpoint of the High School Teacher."

On Saturday evening at 6:30 o'clock, the joint banquet was held for all mathematicians, members of the National Council, Mathematical Association of America, and the American Mathematical Society. Professor Dunham Jackson of the University of Minnesota acted in the capacity of toastmaster and presented to the audience our distinguished guest, Professor Albert Einstein. Professor Jackson called upon several of our own distinguished mathematicians who responded in a happy vein.

One significant result of the meeting at Pittsburgh was the establishing of closer and more cordial relations between the high school and the college teachers of mathematics. We secured several new members of the council from among the college group.

MEETING OF THE BOARD OF DIRECTORS
HOTEL WEBSTER HALL

Saturday, December 29, 1934

PRESENT: J. O. Hassler, Edwin W. Schreiber, W. D. Reeve, William Betz, H. C. Christofferson, Edith Woolsey.

ABSENT: Ralph Beatley, A. R. Congdon, Vera Sanford, H. E. Slaught, John P. Everett, Elsie Parker Johnson, Raleigh Schorling, Harry C. Barber, Martha Hildebrandt, W. S. Schlauch.

Six members of the Board of Directors were present at the luncheon meeting together with Professor Edwin G. Olds, Chairman of the local committee. The Board of Directors expressed its deep appreciation of the work of Professor Olds and his local committee consisting of the following Pittsburgh teachers: Dr. E. B. Cowley, Allegheny High; H. E. Long, Westinghouse High; Sarah B. Houston, Langley High; E. F. M. Patton, Carrick High; J. C. Stuchel, Oliver High; J. H. Bortz, Peabody High; J. A. Silver, South Hills High; Mercedes Schramm, Fifth Avenue High. All the minor details so necessary for the efficient running of a meeting of this sort were very ably taken care of by Professor Olds and his committee. The local committee provided the following entertainment for visiting mathematicians: a tea on Thursday afternoon and a bridge party on New Years Eve. It was moved and carried that the Geometry Report be published as a Supplementary Monograph if possible.

ATTENDANCE

California (1)

Los Angeles: Hedrick, E. R. (R) Univ. of Cal. at L. A.

Connecticut (1)

Hartford: Donchian, Paul S. (G)

District of Columbia (1)

Washington: Scheier, M. A. (R) Cath. Univ.

Illinois (7)

Chicago: Baldwin, Wm. K. (G); Christman, S. E. (W) Senn H. S.; Macknik, Ted (G)

Evanston: Moulton, Elton J. (R) Northwestern U.

Freeport: Martin, Mary C. (W) Freeport H. S.

Macomb: Schreiber, Edwin W. (B–11) State Teachers College

Urbana: Taylor, S. Helen (R–3) University High

Indiana (2)

Bloomington: Davis, H. T. (G) Indiana Univ.; Williams, K. P. (R) U. of Indiana

Iowa (1)

Cedar Falls: Kearney, Dora (R–7) State Teachers College

Kansas (1)

Lawrence: Mitchell, U. G. (R–3) U. of Kansas

Maryland (1)

Frederick: Brown, Lillian (R–1) Hood College

Massachusetts (1)

Williamstown: Richmond, D. E. (G) Williams College

Michigan (2)

Ann Arbor: Van Engen, Henry (G) Tappan

Detroit: Nelson, A. L. (R–2) Wayne Univ.

Minnesota (3)

Minneapolis: Woolsey, Edith (R–3) Sanford Jr. H. S; Hart, W. L. (G) Univ. of Minn.; Jackson, Dunham (R–5) Univ. of Minn.

Missouri (2)

St. Louis: Dietz, Paul (G) The Principia School; Roever, Wm. H. (G) Washington Univ.

New Jersey (6)

Jersey City: Smith, Rev. J. P. (G) St. Peters Col.

Maplewood: Reynolds, C. I.(W) Maplewood Jr. High

Newark: Mandeville, Marjorie (W) Silver Burdette & Co.

New Brunswick: Meder, Albert E. (W) New Jersey

College for Women: Morris, Richard (W) Rutgers Univ.

Summit: Shuttlesworth, Joseph G. (W) Summit High & Bucknell S. S.

New York (16)

Alford: Seidlin, Joseph (R–2) Alfred Univ.

Aurora: Hollcroft, Temple (W) Wells College

Brooklyn: MacNeish, H. F. (R) Brooklyn College

Kenmore: Braunton, Wm. H. (W) Kenmore Sr. High

New York City (8): Atherton, Charles R. (R–2) Columbia; Clifford, Paul C. (W) Sethlow Jr. College—Columbia Univ.; Drushel, J. D. (R–2) N. Y. Univ.; Jablonower, Joseph (R–6) Fieldston School; Loveland, Gilbert (G–1) Henry Holt and Company Inc.; Montgomery, Allen Guy (R–2) Columbia Univ.; Reeve, W. D. (B–14) Teachers College Columbia Univ.; Swensen, John A. (R–2) Teachers College Columbia Univ.

Rochester: Betz, William (B–12) Public Schools; Sister Rose Miriam (G)

Nazareth Col.; Smyth, Sister M. Pauline (R–2) Aquinas Institute

Syracuse: Carroll, J. S. (R–2) Syracuse Univ.

Ohio (7)

Cleveland: Morris, Max (G) Case School

Columbus: MacDuffee, C. C. (G) Ohio State

Kent: Rogers, Harold P. (G) Kent State College

Marietta: Reynolds, Delbert (G) Marietta College

Oberlin: Cairns, W.D. (R–6) Oberlin Col.

Oxford: Christofferson, H. C. (R–5) Miami Univ.

Steubenville: Boyd, Margaret (W) Steubenville H. S.

Oklahoma (1)

Norman: Hassler, J. O. (B–8) Univ. of Okla.

Pennsylvania (78)

Aliquippa: Shank, C. Earl (G) Aliquippa H. S.

Aspinwall: McCrum, Jean (W–1) Aspinwall H. S.

Braddock: George, Clementina (W) Braddock H. S.; Redman, Anna (W) Braddock High

Carnegie: Lauble, Ida (W) Carnegie High

Donora: Tefft, Charles E. (W–1) Sr. High S.

Erie: Wells, E. D. (G) Erie Center

Glassport: Srp, Joseph A. Jr. (G) Glassport High

Greensburg: Bortz, John H. (W) Peabody

Homestead: Gallager, Margaret (G) High; Jose, Alice (W–1) Homestead H. S.; Vallowe, Louise (R) Homestead H. S.; Muir, Mary P. (G) Home stead H. S.

Huntingdon: Shively, Chas. S. (G) Juniata College; Stayer, J. Clyde (G) Juniata College; Belles, Oscar (G) Westmont High School

Monessen: Williams, Helen (W–1) Monessen High

Mt. Lebanon: Rightmire, Anne (W) Mt. Lebanon High

New Castle: Grossman, Harold (G) Geo. Washington Jr.–Sr. High School; McCune, Sara Sankey (R–3) Senior High

Northampton: Buell, Clayton E. (G) Senior H. S.

Pittsburgh (45): Bortman, Edith (G) Winchester School; Campbell, Elizabeth (G) Knoxville School; Cotton, Reardon S. (G) South Hills High School; Cowley, Dr. Elizabeth B. (R–4) Pgh. Public Schools (H.S.); Epley, Isabel (G) Herron Hill; Freeble, C. R. (G) Allderdice; Fretts, L. Ruth (G) Dormont High; Geary, Rosemary (G) David B. Oliver; Glass, J. S. (W) Allderdice H.S.; Horner, R. E. (G) Langley High; Ion, Ella (G) Mt. Lebanon H.S.; Jacobi, Helen (R–2) So. Hills High; Johnson, H. H. (G) South High; Kelso, John S. (G) Latimer Jr.; Kirkpatrick, Bertha (G) Westinghouse.; Kreps, Ida B. (W) Peabody High; Leaman, Theodore (W) Arnold School Private; Leopold, G. A. (G) Westinghouse; Long, Harry E. (W) Westinghouse; Lyons, Catherine A. (W) D. B. Oliver High; McKinney, Katherine M. (W) Dormont High School; Miser, Walter D. (G) Carrick High; Morris, H. T. (G) C. B. Connelley; Olds, Edwin G. (G) Carnegie Tech.; Patton, E. A. M. (G) Carrick H.S.; Paul, Elsie L. (G) Fifth Ave. High; Pickard, Dorothy (W) Taylor Allderdice Jr. High; Porter, Mary Anne (W) Oliver High; Roemhild, E. O. (G) National Tube Co.; Rosenback, Joseph B. (G) Carnegie Inst. of Tech.; Snyder, George B. (G) Taylor Allderdice High; Staley, W. J. (G) Carnegie Tech.; Steele, Donald (G) Westinghouse; Stein, Mary A. (G); Stein, Stella M.

(W) South High; Stuchel, J. C. (W) D. B. Oliver H.S.; Taylor, James S. (G) U. of Pitts.; Townley, Ruth (W) Schenley High; Wagner, Wesley (G) Schenley High; Washburn, S. Merle (G) Oliver High; Wylie, Carrie L. (W) Langley High; Schlegel, Edna (G) Gladstone Jr. High; Schramm, Mercides (G) Fifth Ave. H.S.; Seneff, Mrs. N. L. (R) South Hills High; Silver, James A. (W) South Hills H.S.

Sharpsburg: Geary, Marcella (W) Sharpsburg High

Springdale: Spalic, John (G) Springdale H.S.

State College: Hagen, Beatrice (G) Penn State College; Shileli, (R–2) Penn State College

Swarthmore: Brinkman, H. W. (G) Swarthmore College; Dresden, Arnold (G) Swarthmore College

Swimvale: Jackson, Guy (G) Swimvale High

Trafford: Wieland, R. Waldo (W) Trafford High School

Wampum: Brown, Paul (W) Wampum H.S.

Warren: Lafferty, Wm. (W) Warren High

West View: Paynter, W. R. (G) West View High

Wilkinsburg: Tedrow, Kenneth (G) Wilkinsburg Sr. High School

Rhode Island (2)

Providence: Archibald, H. C. (G) Brown Univ.; Bennett, A. Z. (G) Brown Univ.

Tennessee (1)

Nashville: Wren, Frank L. (R–2) George Peabody College

Wisconsin (2)

Madison: Hartung, M. L. (R–4) University High

W. De Pere: DeCleeve, Rev. L. A. (W) St. Norbert Col.

ATTENDANCE BY STATES

	G.	W.	R.	B.	Total
1. California	0	0	1	0	1
2. Connecticut	1	0	0	0	1
3. Dist. of Col.	0	0	1	0	1
4. Illinois	2	2	2	1	7
5. Indiana	1	0	1	0	2
6. Iowa	0	0	1	0	1
7. Kansas	0	0	1	0	1
8. Maryland	0	0	1	0	1
9. Massachusetts	1	0	0	0	1
10. Michigan	1	0	1	0	2
11. Minnesota	1	0	2	0	3
12. Missouri	2	0	0	0	2
13. New Jersey	1	5	0	0	6
14. New York	2	4	8	2	16
15. Ohio	4	1	2	0	7
16. Oklahoma	0	0	0	1	1
17. Pennsylvania	46	27	5	0	78
18. Rhode Island	1	1	0	0	2
19. Tennessee	0	0	1	0	1
20. Wisconsin	−0	1	1	0	2
TOTAL	63	41	28	4	136

CODE: G.—Visitor or Guest
W.—Member attending first meeting
R.—Member attending 2–7 meetings
B.—Member attending 8 or more meetings

THE Illinois Division of the National Council of Teachers of Mathematics was called to order by the chairman, Mr. William Clark, in room 300, Mathematics Building, on November 23, 1934, at 9:10 A.M.

Mr. Clark appointed the following committee to nominate one member to replace the retiring member of the executive committee of the Illinois Division, and a delegate, and an alternate to the Atlantic City meeting of the National Council:

Dr. E. B. Lytle, University of Illinois, Urbana, Chairman

Mr. J. T. Johnson, Chicago Normal, Chicago

Mr. Clarence Fuqua, Champaign High School, Champaign.

Miss Alvena Bamberger of the Champaign High School presented a paper, "Some Problems in the Teaching of Ninth Grade Algebra." At its conclusion questions were asked and a discussion was carried on by the members.

Mr. E. W. Schreiber, secretary of the National Council, spoke to the group concerning membership on the National Council, which includes subscription to *The Mathematics Teacher*. He also recommended the yearbooks of the council.

Dr. Helen Taylor of the University High School also urged teachers of mathematics to become affiliated with the National Council.

Prof. J. A. Clement, College of Education, University of Illinois, presented a paper, "Interrelationship Between the Purposes, Content and Teaching Technique of Ninth and Tenth Grade Mathematics." This was followed by a discussion of the necessity of the reorganization of mathematics.

The meeting then adjourned for the noon recess.

The meeting was again called to order at about 2:00 P.M., Mr. William Clark presiding.

Dr. E. B. Lyttle, Chairman of the nominating committee, presented the following report:

We wish to nominate Prof. C. N. Mills, Illinois State Norman University, Norman, Illinois, for election to the executive committee to serve three years, to act as secretary of the executive committee next year.

We recommend that Miss Martha Hildebradt, next year's chairman of the executive committee be empowered to appoint the delegate of this section to the New Jersey meeting of the National Council of Mathematics Teachers.

Nominating Committee { J. T. Johnson, Clarence Fuqua, E. B. Lytle, Chairman

This left the following committee: Miss Martha Hildebrandt, Provisio Township High School, Maywood, Chairman; Mr. E. G. Hexter, Township High School, Belleville, Vice-Chairman; Prof. C. N. Mills, Illinois State Normal, Secretary. It was moved, seconded and voted that the report be adopted as read.

Miss Clara D. Murphy, of the Evanston High School, presented a paper, "Special Courses for Low Ability Students." A discussion followed.

Mr. C. N. Austin of the Oak Park High School, recommended membership in the National Council and ownership of the yearbooks to high school teachers of mathematics. He also announced the annual meeting of the Central Association of Science and Mathematics Teachers in Indianapolis the following week.

Mr. Herbert O. Barnes, of the Springfield High School, presented a paper," "Analytical Expression in Formal Demonstration." This was followed by a spirited discussion in which many members participated.

The meeting adjourned.

E. G. Hexter
Secretary

THE FALL meeting of the Connecticut Valley Section of the Association of Teachers of Mathematics in New England was held on Saturday, November 3, 1934, at Central High School in Springfield, Massachusetts.

MORNING SESSION

10:00 Social gathering in Room 116.

10:15 "Mathematics and the Problem of the Absolute."
Mr. Francis C. Bailey,
Mount Hermon School

10:30 "This Simple Universe."
Professor Bancroft H. Brown
Dartmouth College

"The New Method of Grading and Listing Candidates in Algebra by the College Entrance Examination Board."
Professor William Raymond Langley
Yale University

12:30 Luncheon.

AFTERNOON SESSION

1:45 Business Meeting.
2:00 "Number Sense."
Miss Marion Bills
Assistant Secretary
Aetna Life Insurance Co.
"Simple Continued Fractions."
Professor William Fitch Cheney, Jr.
Connecticut State College

OFFICERS OF THE CONNECTICUT VALLEY SECTION

Professor David D. Leib, *Vice President**
Connecticut College, New London

* Mr. Caroll G. Ross, President-elect, was unable to accept because of removal from the territory.

Miss Helen Wright,.....*Secretary*
Northfield Seminary
Miss Beatrice Neal,.....*Treasurer*
Bulkeley High School, Hartford
Professor H. M. Dadourian,...*Director*
Trinity College, Hartford
Mr. Fred L. Mockler,.....*Director*
Holyoke High School

PAST PRESIDENTS

Harry B. Marsh	'20
Percey F. Smith	'21
M. M. S. Moriarty	'22
Eleanor C. Doak	'23
Joe G. Estill	'24
Joshua I. Tracey	'25
Lyon L. Norton	'26
John W. Young	'27
Rolland R. Smith	'28
Harriet R. Cobb	'29
Melvin J. Cook	'30
Bancroft H. Brown	'31
Dorothy S. Wheeler	'32
Haroutune M. Dadourian	'33

AT THE mathematics section of the Conference on Education at Bucknell University on Saturday November 3rd, 1934, the following program was given:

Problems in the Junior High School Mathematics Curriculum.—Genevieve M. Punches, Theodore Roosevelt Junior High School, Williamsport, Pa.

Some Current Trends in Secondary Mathematics—Joseph G. Shuttlesworth, Department of Mathematics, High School, Summit, N. J.

Chairman, Robert D. Smink, Department of Mathematics, Senior High School, Williamsport, Pa.

THE FOLLOWING program was given at the annual meeting of the Association of Teachers of the Middle States at Haddon Hall in Atlantic City, N. J. on Saturday morning December 1, 1934.

President—CECIL A. EWING, Tome School, Port Deposit, Md.

Secretary—RUTH WYATT, Woodrow Wilson Junior High School Philadelphia.

9:30 A.M. Meeting of the Council—Green Room, Meeting Room Floor

10:30 A.M. Meeting of the Association—Room 1344, Haddon Hall, Meeting Room Floor

Business Meeting: Reports. Election of Officers

Address—Why Do We Teach Mathematics?: ARNOLD DRESDEN, Professor Mathematics and Astronomy, Swarthmore College

Address—Ninth Year Mathematics for Non-College Pupils. ROSCOE P. CONKLING, Head Mathematics Dept., Central High School, Newark, N. J.

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Topics of interest for members of the group will be: The new curricula in mathematics in various schools and universities, mathematics instruction, mathematics in trade and vocational schools, museums of special value to teachers of mathematics and science, lectures and discussions on mathematics teaching, and training of mathematics teachers. Such cities as Hamburg, Göttingen, Berlin, Dresden, Munich, Frankfurt, Cologne, Düsseldorf and Essen will be included. Provision will also be made for some sight-seeing and entertainment.

The group will sail from New York about June 20 and will sail from Hamburg for the United States on August 10. The total cost including passage from New York to New York will be $550.00, to cover all necessary expenses except tips on the boat, passport, personal expense, and amusements. This cost includes the University fee and four points of tuition. Additional points of credit is at the rate of $10.00 a point.

For further information address:

Professor W. D. Reeve
Teachers College, Columbia University
525 West 120th Street, New York City

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Informational Arithmetic. By B. R. Buckingham.

The Relation of Social Arithmetic to Computational Arithmetic. By G. T. Buswell of University of Chicago.

Opportunities for the Use of Arithmetic in an Activity Program. By Paul R. Hanna of Lincoln School of Teachers College, Columbia University, and others.

Economy in Teaching Arithmetic. By J. T. Johnson of Chicago Normal College.

Current Practices in Teacher-Training Courses in Arithmetic. By Romie Dustin Judd of Morehead State Teachers College, Morehead, Ky., and Robert Lee Morton of College of Education, Ohio University, Athens, Ohio.

The Problem of Transfer in Arithmetic. By James Robert Overman of Bowling Green State College, Bowling Green, Ohio.

Types of Drill in Arithmetic. By Austin C. Repp of College of Education, University of Arizona.

Retrospect, Introspect, Prospect. By David Eugene Smith, Emeritus Professor, Teachers College, Columbia University.

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